Use the Comparison Table for Term and Whole Life Insurance to find the yearly premium for the amounts of insurance listed below at the ages shown.

	Amount of Insurance	Age	Type	Premium
14.	$ 50,000	30	Term	
15.	50,000	30	Whole life	
16.	150,000	40	Term	
17.	150,000	40	Whole life	
18.	250,000	50	Term	
19.	250,000	50	Whole life	

20.–22. For each of the three pairs of policies (term and whole life) in Exercises 14–19, which is the less expensive? How much cheaper is the less expensive choice?

MIXED REVIEW

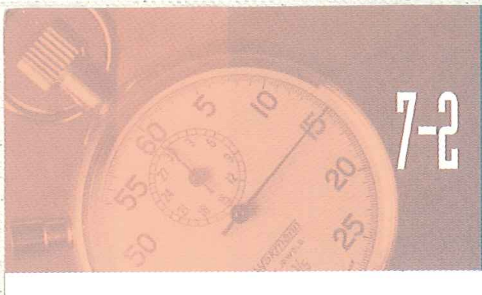
Assume that a person receives $1\frac{1}{2}$ times the usual hourly rate for each hour or part of an hour beyond 40 hours per week. Find the total wages earned in one week in each case.

1. Hourly wage rate: $8.25; total hours worked: 50
2. Hourly wage rate: $11.00; total hours worked: $47\frac{1}{2}$
3. Suppose that you can pay off a $12,500 loan either by paying $395 per month for 3 years or by paying $260 per month for 5 years. How much will you save if you pay the loan off in 3 years?

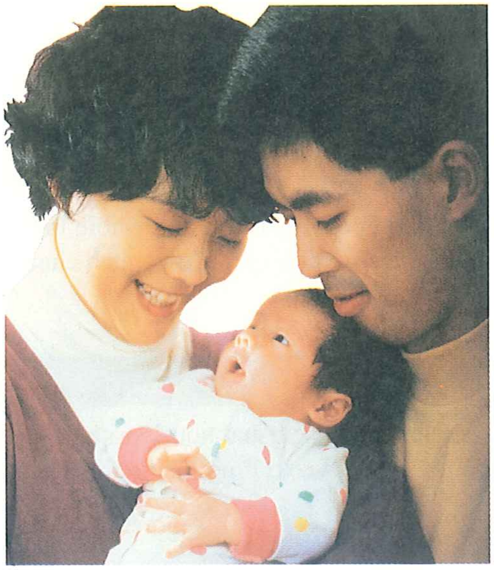
Complete the chart below to determine how much the interest cost will be if you pay 10% of the amount due to the nearest dollar each month and your credit card carries an APR of 9%.

	Month	Balance	Interest	Amount Owed	Payment
4.	1	$880.00			
5.	2				

6. What is the total interest for the two months?
7. Tom's weekly gross pay is $115. The amount withheld for taxes is $2 and 7.65% of the gross pay is withheld for FICA taxes. Find Tom's take-home pay for the week.
8. Write an inequality for each of the following 3 constraints: you want to work no more than 15 hours per week, you want to earn at least $100 a week, and you won't get a job paying more than $9 an hour. Let x represent the number of hours you work in 1 week and y represent your weekly salary.



7-2 SPREADING THE RISK: HOW INSURANCE WORKS



Martin and Rachel finally decided to purchase term insurance on Martin's life rather than the other types of life insurance that were available. Since Martin is still in his twenties, he was able to get a policy with a relatively low premium. The oldest brother, Benjamin, has also started a family recently. Because he is in his thirties, the term insurance that Benjamin purchased is almost $50 more per year than Martin has to pay for the same $100,000 policy. Manuel knew that insurance premiums increase as the age of the insured person increases, but he wondered why the amount of the increase was $50. Why not a $5 increase—or $500?

Lily has recently had a similar experience. Her father told her that he had rejected an insurance salesman's argument that Lily's life should be insured "because her premiums would be so low." She and her father both know that she doesn't need life insurance because no one depends on her for support. Nevertheless, like Martin, she wants to know why her premiums would be so low.

Manuel decided to talk with his cousin Roberto, who works for a major insurance company. One of Roberto's duties is to help prepare new tables that show the premiums that people have to pay for insurance. Manuel hoped that Roberto could explain to him and Lily how the numbers in the tables are computed.

OBJECTIVES: *In this lesson, we will help Lily and Manuel to:*

- *Understand how life-expectation tables are used to estimate the probability that an individual will die within one year.*
- *Learn how an insurance company determines its premium schedule to make a reasonable profit.*

STATISTICAL TABLES

Manuel and Lily visited Roberto at his office. As they were talking, Manuel noticed that on Roberto's desk was a statistical table entitled Expected Deaths per 100,000 Alive at Specified Age. Part of the table is shown below. This table is also in the Reference Section.

EXPECTED DEATHS PER 100,000 ALIVE AT SPECIFIED AGE		
Age	Expected Deaths Within 1 Year	Expected to be Alive in 1 Year
15	63	99,937
16	79	99,921
17	91	99,909
18	99	99,901
19	103	99,897
20	106	99,894
21	110	99,890
22	113	99,887
23	115	99,885
24	117	99,883
25	118	99,882
26	120	99,880
27	123	99,877
28	127	99,873
29	132	99,868

Roberto explained that information in the table enabled him to calculate the probability that a person of a given age will die or not die sometime during the next 12-month period. This information helps him to decide how large a life insurance premium should be for each age.

For example, the probability that a 16-year-old person will still be alive 1 year from today is found by using the formula for the **probability of an event.** This probability is written as $P(E)$, which is read "P of E."

Probability of an Event

$P(E) = \frac{m}{n}$ where $P(E)$ = the probability of an event E
m = the number of times the event occurs
n = the number of all possible outcomes

Recall that the probability of an event is always a number between 0 and 1, inclusive.

From the chart, Roberto is able to determine the probability that a 16-year-old person will be alive 1 year from today.

$$P(E) = \frac{99{,}921}{100{,}000} \qquad \frac{\text{Number of 16-year-old people alive 1 year later}}{\text{Total number of 16-year-old people}}$$
$$= 0.99921$$

The probability that a 16-year-old will be alive 1 year from today is 0.99921, that is, almost 1. The event E is practically certain.

The probability that a 16-year-old will die within one year is

$$P(E') = \frac{79}{100{,}000}$$
$$= 0.00079$$

This probability is 0.00079, or almost zero. The event E' ("E prime") is very unlikely.

In the above formula, E' is the event "E does not occur," or "not E." For this reason it is called the **complement** of E. Notice that

$$P(E) + P(E') = 0.99921 + 0.00079$$
$$= 1.00000$$

In words, *the sum of the probabilities of an event and its complement is 1.* For example, it is certain (a probability of 1) that a 16-year-old will either be dead or alive 1 year from today.

Manuel found the discussion about probabilities very interesting, but he still did not have an answer to his question: How is the size of an insurance premium determined?

Ask Yourself

1. How do you think insurance companies can obtain information about life expectancy?
2. If a person's life expectancy is very high, does that mean that he or she can safely drive without a seat belt at high speeds? Explain your answer.
3. In earlier centuries, life expectancy for infants and very young children was much lower than it is today. Why do you think that the life expectancy for children has improved in recent years?

ALGEBRA REVIEW

At Central High there are 240 juniors. Of these, 110 are girls, 180 study math, 60 take Spanish, 40 take French, and 100 go to first lunch. No students take both Spanish and French. A student's name is to be drawn for a prize. Each name has an equal chance of being drawn.

Find the following probabilities to the nearest thousandth for the student whose name is selected.

1. Be a girl
2. Study Spanish
3. Study French
4. Study either Spanish or French
5. Study neither Spanish nor French
6. Go to first lunch
7. Not go to first lunch
8. Be a boy
9. Study both Spanish and French
10. Win the prize

SHARPEN YOUR SKILLS

SKILL 1

The **expected value** is the amount of money to be won or lost in the long run. If an event can assume two values, then the *expected value* of the event is the sum of the product of each value and its probability.

Expected Value Formula

$E = P_1v_1 + P_2v_2$ where v_1 and v_2 are values and P_1 and P_2 are the corresponding probabilities

EXAMPLE 1 In a certain game of chance, you win \$4 when a coin shows heads and \$1 when it shows tails.

QUESTION What is the expected value of money that you could win?

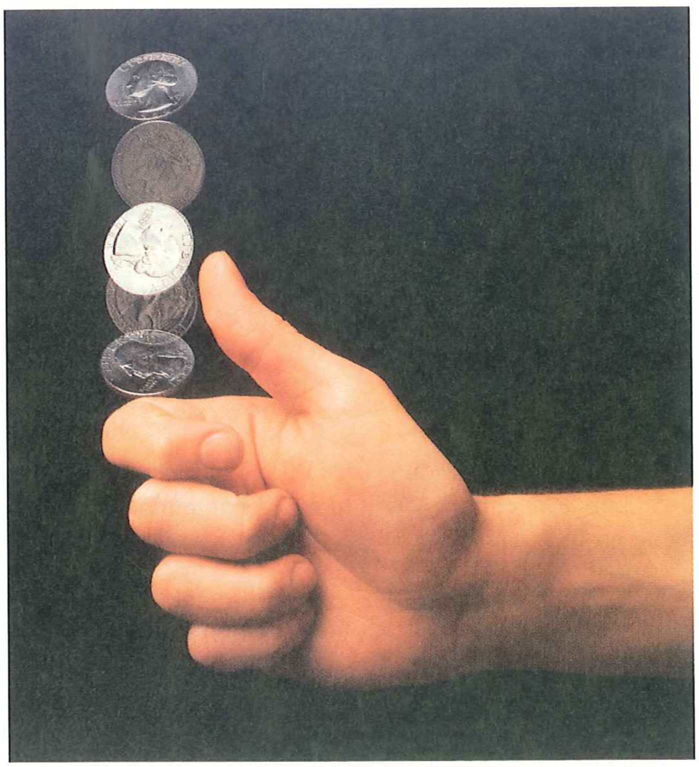
SOLUTION

The probability of obtaining heads and the probability of obtaining tails are each 0.5.

$E = P_1v_1 + P_2v_2$ Expected value formula

$E = 0.5(4) + 0.5(1)$ $v_1 = 4$, $v_2 = 1$, $P_1 = 0.5$, $P_2 = 0.5$

$= 2.50$



For the "privilege" of tossing the coin you should pay no more than $2.50. Notice that on a single toss you will win either $1 or $4, never $2.50. For a large number of tosses you can expect to gain about $2.50.

SKILL 2

Roberto's insurance company must make a reasonable profit on the insurance policies that it writes. Otherwise, it will not be able to pay benefits on the policies or even to survive as a company. Roberto uses the idea of expected value to find the premium that gives the **break-even value** for the insurance company; that is, the value of the premium that gives zero profit after paying for all expenses.

EXAMPLE 2 The direct and indirect expenses for each policy that the insurance company writes are about $20 per policy.

QUESTION How can Roberto use probability to determine the proper premium for a $50,000 insurance policy on a 28-year-old person?

SOLUTION

The possible outcomes in the next year for a 28-year-old person are as shown in the table on page 331. The variable x represents the break-even value for the premium. Notice that the probability that a person dies is 1 minus the probability that a person lives.

COMPANY'S GAIN OR LOSS ON ONE POLICY (before deducting expenses)		
Possible Outcome	**Probability of Outcome**	**One-year Gain/Loss**
The person lives.	$\frac{99{,}873}{100{,}000}$, or 0.99873	x (gain)
The person dies.	$\frac{127}{100{,}000}$, or 0.00127	$x - 50{,}000$ (loss)

$$E = P_1v_1 + P_2v_2 \qquad \text{Use the expected value formula.}$$
$$E = 0.99873x + 0.00127(x - 50{,}000)$$
$$0 + 20 = 0.99873x + 0.00127x - 63.5 \qquad \text{Profit: 0; expenses: 20}$$
$$83.5 = 1.00000x$$
$$x = 83.5$$

The break-even premium for one year of life insurance on a 28-year-old person is $83.50. The company will have to charge more than $83.50 to make a reasonable profit.

Roberto explained to Manuel and Lisa that this example applies to a *one-year term* life insurance policy, not to a whole life policy. A whole life policy has additional savings features that would raise the premiums significantly. He also mentioned that most term policies are for five or ten years, not one year, and that five-year and ten-year life expectation tables can be used to prepare premium schedules for such policies.

SKILL 3

The profit that an insurance company makes on policies is determined by the amount of revenue (money) that is received as premiums minus any benefits paid out and any other expenses such as overhead.

Profit on Insurance

$P = R - B - C$ where P = profit
R = revenue received as premiums
B = benefits paid out
C = costs or expenses

EXAMPLE 3 Roberto told Lisa and Manuel that his company charges a 20-year-old person \$110 for a one-year term \$50,000 life insurance policy.

QUESTION What profit does the company expect to make on each of 100,000 such policies if the expenses for each policy are \$20?

SOLUTION

Let x represent the profit for each policy. Then $100{,}000x$ represents the profit for 100,000 policies. Use the table of expected deaths on page 331. There are 106 expected deaths for 20-year-old people. The death benefit is \$50,000. So, the benefits paid out are $B = 106(50{,}000)$. The cost is \$20 per policy, so $C = 20(100{,}000)$. The revenue is \$100 per policy, so $R = 110(100{,}000)$.

$$
\begin{aligned}
P &= R - B - C && \text{Use the profit on insurance formula.}\\
100{,}000x &= (110)100{,}000 - 106(50{,}000) - 20(100{,}000)\\
100{,}000x &= 11{,}000{,}000 - 5{,}300{,}000 - 2{,}000{,}000\\
100{,}000x &= 3{,}700{,}000\\
x &= 37 && \text{Divide each side by 100,000.}
\end{aligned}
$$

The company will make \$37 profit on each policy and 37(100,000) or \$370,000 on 100,000 policies.

EXAMPLE 4 To use the table of expected deaths on page 331 for another amount of people, write a proportion.

QUESTION What is the number of expected deaths for 5000 20-year-olds?

SOLUTION

From the table on page 331 the expected deaths for 100,000 20-year-olds is 106. Let d represent the expected deaths for 500 20-year-olds. Then

$$\frac{106}{100{,}000} = \frac{d}{5000} \qquad \text{Multiply both sides by 5000.}$$

$$\frac{106(5000)}{100{,}000} = d$$

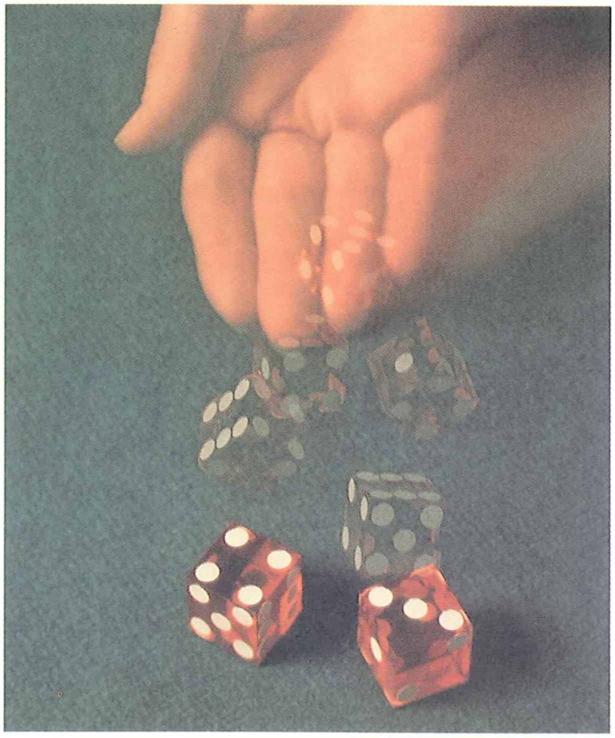
$$5.3 = d$$

The expected deaths for 5000 20-year-olds is 5.3.

TRY YOUR SKILLS

Find the expected value.

1. The expected gain from a coin toss that pays \$3 for heads and \$2 for tails
2. The expected gain from a roll of a die that pays \$10 for a 6 or a 2 and \$1 for any other result



A person's life expectancy for one year is shown in the following table. She is contemplating the purchase of a one-year term policy with a face value of \$60,000. Refer to the table to answer Exercises 3–5.

COMPANY'S GAIN OR LOSS ON ONE POLICY (before deducting expenses)		
Possible Outcome	Probability of Outcome	One-Year Gain/Loss
The person lives.	0.99906	x
The person dies.	?	? – ?

3. Find the probability that the person will die within one year.
4. What is the algebraic expression that represents the company's one-year gain or loss (before expenses) if the person dies?
5. What is the algebraic expression that represents the company's expected gain or loss (before expenses) on the policy?
6. The company has direct and indirect expenses of \$25 for each policy that it issues. Find the premium that the company must charge to break even; that is, neither to make nor to lose money on this policy.

Use the formula for profit on insurance $P = R - B - C$ and the table of expected deaths to calculate the profit that a company makes on one-year term life insurance policies for the policies and populations described in Exercises 7–10. Begin by letting x represent the money made on each policy to cover profit. The cost for each policy is \$25.

7. 1000 19-year-olds; face value: \$100,000; annual premium: \$200
8. 1000 23-year-olds; face value: \$40,000; annual premium: \$100
9. 5000 28-year-olds; face value: \$135,000; annual premium: \$350
10. 50,000 25-year-olds; face value: \$45,000; annual premium: \$125

EXERCISE YOUR SKILLS

1. Why do you think that the probability of a 16-year-old being alive one year from now is higher than that of a 25-year-old?
2. Why don't insurance companies charge the same life insurance premium for everyone regardless of his or her age?
3. Is there any age group for which an insurance company could not issue a life insurance policy? Explain your answer.
4. Suppose that two insurance companies both sell five-year term life insurance policies but that one of the companies charges an annual premium that is $30 higher than the other company's. What are some possible explanations for this?
5. From time to time, insurance companies update the tables that they use to help them set premium schedules. Why do you think that they have to update the tables?
6. Because they deal with large populations of people, life insurance companies can use principles of probability to help them set profitable premium schedules. Does this mean that an insurance company can never suffer a loss from selling insurance policies? Explain your answer.

A winning lottery ticket pays $200. Find the expected value of the lottery for each number of sold tickets.

7. 50 tickets
8. 100 tickets
9. 500 tickets
10. In a certain store the probability that a person makes one purchase is 0.7; the probability that a person makes two purchases is 0.3. Find the expected number of purchases.

Find the break-even premium for a one-year term insurance policy for each indicated individual. In each case, assume that the direct and indirect expenses for issuing one policy are $30. Use the table of expected deaths on page 331.

11. a $50,000 policy for a man or woman of 24
12. a $100,000 policy for a man or woman of 29
13. Find the break-even premium for a one-year term insurance policy for $200,000 for a man or woman of 35. The insurance company expects 99.825% of all 35-year-old people to live at least one more year.
14. Find the break-even premium for a one-year term insurance policy for $150,000 for a man or woman of 38. The insurance company expects 99.765% of all 38-year-old people to live at least one more year.
15. Find the break-even premium for a one-year-term insurance policy for $250,000 for a man or woman of 39. The insurance company expects 99.725% of all 38-year-old people to live at least one more year.

KEY TERMS

break-even value
complement
expected value
probability of an event

What profit does an insurance company expect to make for each one-year term insurance policy? Assume that the direct and indirect expenses for each policy are $25.

16. Face value: $50,000; age of insured: 25; annual premium: $135

17. Face value: $150,000; age of insured: 22; annual premium: $390

18. Face value: $290,000; age of insured: 26; annual premium: $800

19. Face value: $250,000; age of insured: 29; annual premium: $650

20 Face value: $75,000; age of insured: 24; annual premium: $200

21. Face value: $125,000; age of insured: 27; annual premium: $325

An insurance company employee has the responsibility of determining the company's schedule of life insurance premiums. She is reviewing the following portion of a life-expectancy table.

EXPECTED DEATHS PER 100,000 ALIVE AT SPECIFIED AGE		
Age	**Expected Deaths Within 1 Year**	**Expected to be Alive in 1 Year**
45	315	99,685
46	341	99,659
47	371	99,629
48	405	99,595
49	443	99,557

The company's marketing department has told her that the public loses interest in buying life insurance whenever the annual premium is greater than $800. The company's current expenses for each policy issued are $35 and it aims at making a profit of $50 on each policy.

22. At what age will a person first be inclined to resist purchasing one-year term life insurance with a face value of $200,000?

23. To appeal to the person of Exercise 22, the company manages to reduce its expenses to $20 per policy. Will this be enough to enable the employee to lower the premium to $799 without cutting into the company's profits? If not, how much of a reduction in profits would the company have to accept for the employee to be able to lower the premium to $799?

24. The company wants to sell a one-year term policy to a 49-year-old person and keep the premium below $800. Assume that the expenses have been reduced to $20 per policy and that the profit on the policy has been raised to $55. What would be the largest possible face value of such a policy?

25. The company wants to sell a one-year term policy to a 45-year-old person and keep the premium below $500. Asume that the expenses are $20 per policy and that the profit on the policy is $60. What would be the largest possible face value of such a policy?

MIXED REVIEW

Suppose that you write a check for the amount shown. Write the amount in words.

1. $500
2. $10,100.10
3. Which approach will save you more money; to make a 20% down payment on a purchase, or to save up until you can afford a 30% down payment? Explain your answer.
4. Determine the effective rate of interest if the APR is 14.75%.

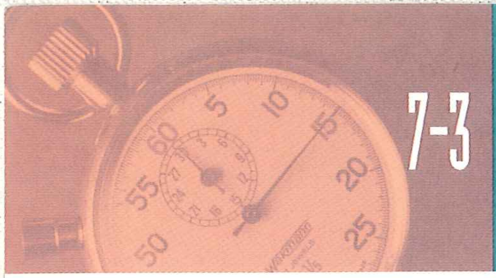
Complete the following chart to determine how much the interest cost will be if you pay 5% of the amount due to the nearest dollar each month and your credit card carries an APR of 9%.

	Month	Balance	Interest	Amount Owed	Payment
5.	1	$1000.00			
6.	2				

7. What is the total interest for the two months?
8. What happens to the nation's money supply when the Federal Reserve System increases the percent of required reserves that banks must hold?
9. You earn $7.25 per hour. You worked 46 hours last week. You receive $1\frac{1}{2}$ times your hourly rate for any hours over 40. Find your total wages for last week.
10. Last month the Wilson family spent $205 on the car payment, $506 on credit card payments, $850 on utilities and rent, $560 on food, and $2,000 on everything else. To the nearest tenth of a percent, what percent of their take-home pay did the family spend on credit payments?
11. Use the Time-to-Pay-Off formula to find the number of months required to pay off a credit card balance of $1,850. The APR is 15% and the monthly payment is $120.
12. You have a loan of $25,000 at 10.5% for 5 years for which you have been making monthly payments of $537.35. How much money will you save if you prepay the loan at the end of 40 months?
13. Suppose that a bank's reserve requirement is 25%. How much new money can be created from a deposit of $2000?

Some students purchased a number of damaged tote bags for $1.00 each and repaired them for $0.50 each. They plan to sell the repaired tote bags.

14. What is the unit cost for purchasing and repairing the tote bags?
15. Find the total cost for 56 tote bags.
16. The fixed costs are $62. What are the total costs?



7-3 VALUE FOR THE FUTURE

While Lily and Manuel are taking a look at life insurance, their friend Eleanor has other concerns—the financial well-being of her grandparents. Her grandmother, who retired earlier this year, has had to cut back on some expenses to help her family make ends meet.

Eleanor's grandfather retired three years ago. Since then, he has had some costly health problems that are not completely covered by Medicare. In addition to dealing with these past cost burdens, both grandparents have had to purchase supplementary health insurance to cover some of their future medical expenses that will not be covered by Medicare.

Eleanor is not certain of the details, but she does know that the retirement pensions that her grandparents receive are just enough to keep them comfortable. She also knows that her grandparents have had a little difficulty adjusting to the fact that they do not have as much income as they did when they were both working. Eleanor's mother is ready to help her parents in case of a financial emergency but hopes that she will not have to step in. Their circumstances have caused her to begin reevaluating her own retirement plans, and even Eleanor has suddenly become interested in topics such as annuities and Individual Retirement Arrangements.