

## Study Guide

### Graphing Other Trigonometric Functions

The period of functions  $y = \csc k\theta$  and  $y = \sec k\theta$  is  $\frac{2\pi}{k}$ , where  $k > 0$ . The period of functions  $y = \tan k\theta$  and  $y = \cot k\theta$  is  $\frac{\pi}{k}$ , where  $k > 0$ . The phase shift and vertical shift work the same way for all trigonometric functions. For example, the phase shift of the function  $y = \tan(k\theta + c) + h$  is  $-\frac{c}{k}$ , and its vertical shift is  $h$ .

#### Example 1 Graph $y = \tan x$ .

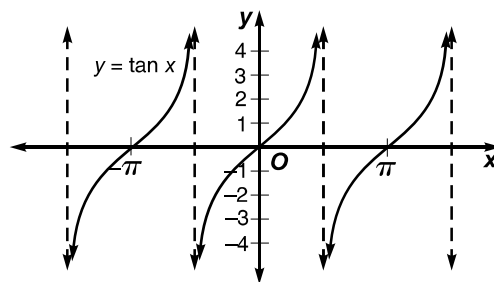
To graph  $y = \tan x$ , first draw the asymptotes located at  $x = \frac{\pi}{2}n$ , where  $n$  is an odd integer. Then plot the following coordinate pairs and draw the curves.

$$\left(-\frac{5\pi}{4}, -1\right), (-\pi, 0), \left(-\frac{3\pi}{4}, 1\right), \left(-\frac{\pi}{4}, -1\right),$$

$$(0, 0), \left(\frac{\pi}{4}, 1\right), \left(\frac{3\pi}{4}, -1\right), (\pi, 0), \left(\frac{5\pi}{4}, 1\right)$$

Notice that the range values for the interval  $-\frac{3\pi}{2} \leq x \leq -\frac{\pi}{2}$  repeat for the intervals  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$  and  $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ .

So, the tangent function is a periodic function with a period of  $\frac{\pi}{k}$  or  $\pi$ .



#### Example 2 Graph $y = \sec(2\theta + \pi) + 4$ .

Since  $k = 2$ , the period is  $\frac{2\pi}{2}$  or  $\pi$ . Since  $c = \pi$ , the phase shift is  $-\frac{\pi}{2}$ . The vertical shift is 4.

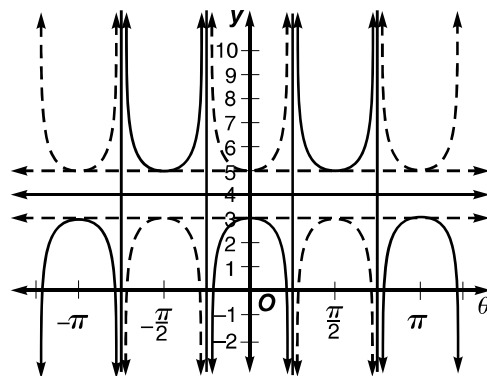
Using this information, follow the steps for graphing a secant function.

**Step 1** Draw the midline, which is the graph of  $y = 4$ .

**Step 2** Draw dashed lines parallel to the midline, which are 1 unit above and below  $y = 4$ .

**Step 3** Draw the secant curve with a period of  $\pi$ .

**Step 4** Shift the graph  $\frac{\pi}{2}$  units to the left.



# Practice

## Graphing Other Trigonometric Functions

Find each value by referring to the graphs of the trigonometric functions.

1.  $\tan\left(-\frac{3\pi}{2}\right)$

2.  $\cot\left(\frac{3\pi}{2}\right)$

3.  $\sec 4\pi$

4.  $\csc\left(-\frac{7\pi}{2}\right)$

Find the values of  $\theta$  for which each equation is true.

5.  $\tan \theta = 0$

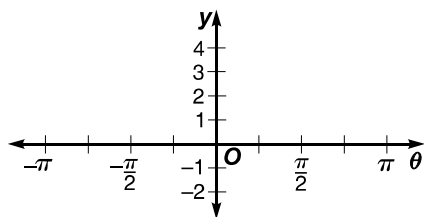
6.  $\cot \theta = 0$

7.  $\csc \theta = 1$

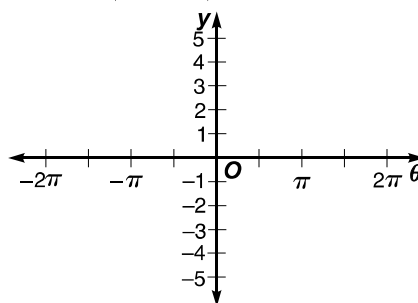
8.  $\sec \theta = -1$

Graph each function.

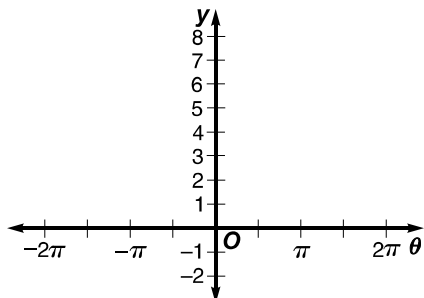
9.  $y = \tan(2\theta + \pi) + 1$



10.  $y = \cot\left(\frac{\theta}{2} - \frac{\pi}{2}\right) - 2$



11.  $y = \csc \theta + 3$



12.  $y = \sec\left(\frac{\theta}{3} + \pi\right) - 1$

