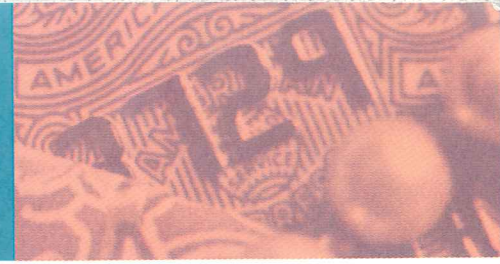# 6-2 CREDIT CARD BALANCES: HOW LONG DO THEY LAST?

Trevor grew up next door to Sylvia and her mother. As kids, he and his younger sister Tracey spent a lot of time playing with Sylvia. Sylvia always wanted to pretend that they were a family so that she could take Tracey shopping for new clothes and toys. Trevor's mother gave them some outdated credit cards with which to play. (She cut out a small portion of each one so that they could not be used to make real purchases.) Sylvia would pull out credit cards to pay for all the imaginary purchases that she made for Tracey.

Since Sylvia had always seen her father with his credit cards, she assumed that everyone made all their purchases with credit cards.

Trevor realized even then how much Sylvia enjoyed shopping on credit. His parents have always had their share of credit cards too, but they do not use them as much as Sylvia's father does. Trevor would like to identify the advantages and disadvantages of credit cards, since Sylvia's father and his parents have different views on their use.

OBJECTIVES: *In this lesson, we will help Trevor to:*

- *Identify advantages and disadvantages of credit cards.*
- *Determine how long it takes to pay off a credit card balance if a fixed payment is made each month.*
- *Determine the size of the monthly payment that must be made in order to pay off a credit card balance in a fixed amount of time.*

ADVANTAGES OF CREDIT CARDS

1. Credit cards are very convenient and simple to use when making purchases.
2. You can delay payment for purchases until a predetermined date.
3. A record of your purchases is made automatically.
4. The danger of losing money while shopping is minimized, since you do not need cash at the time of your purchase.
5. It is easy to order merchandise by mail or telephone with a credit card.
6. Salespeople and business owners may come to recognize you if you are a frequent charge customer and may thus provide you with better service.
7. You can make several purchases at one time.
8. Credit cards help you to take advantage of sales.
9. When you have a credit card, you can use it to obtain an immediate cash advance (up to a specified amount) at an automated teller machine (ATM).

DISADVANTAGES OF CREDIT CARDS

1. A credit card encourages people who have a tendency to overspend to charge beyond their income and ability to pay.
2. Using a credit card does not have the same impact as paying cash. You may forget how much you have charged until the bill comes at the end of the month.
3. Since you can make so many charges on the telephone and through the mail, anyone who knows your credit card number, your name, and other information that is easily obtainable can make unauthorized purchases to your account without having your card in their possession.
4. Credit cards can be stolen. If yours are stolen, you should notify the issuing company immediately, or you could be held liable for unauthorized charges. In general, you will not be liable for unauthorized charges once you have notified the company.

ALGEBRA *REVIEW*

Find the domain and range of each relation.

1. {(9, 2), (3, 8), (9, 8)}

2. {(2, 5), (3, 5), (6,6)}

3. Is the relation in Exercise 1 a function? Why or why not?

4. Is the relation in Exercise 2 a function? Why or why not?

ASK YOURSELF

1. What are four advantages of credit cards?

2. What are three disadvantages of credit cards?

3. If you lose your credit card, why should you immediately notify the company that issued it?

SKILL 1

In this lesson you will use the **common logarithm function,** $y = \log_{10} x$. For this function, y represents the exponent to which the number 10 must be raised to obtain x. For example, $\log_{10} 100 = 2$ because $10^2 = 100$, and $\log_{10} 100{,}000 = 5$ because $10^5 = 100{,}000$. When writing the common logarithm $y = \log_{10} x$, it is customary to omit the base 10. Thus $\log x = \log_{10} x$.

Your graphing calculator has a special common logarithm key. Evaluate $y = \log 1{,}000{,}000$ by pressing the log key on your calculator followed by 1,000,000. Then press the enter key to obtain 6.

A special formula allows you to calculate the length of time that it will take to pay the remaining balance on a credit card if a specified monthly payment M is made. The formula uses the common logarithm function.

Time-to-Pay-Off Formula

$$n = \frac{\log\left(\dfrac{M}{M - Pr}\right)}{\log(1 + r)}$$

where $P =$ amount of loan
$r =$ monthly interest rate
$M =$ monthly payment
$n =$ number of payment periods

EXAMPLE 1   Trevor's father has a balance of $1200 on one of his MasterCard cards that has an APR of 15%. He can afford monthly payments of $150.

QUESTION   If he makes no further purchases with this card, how long will it take him to pay off the balance?

SOLUTION

$$n = \frac{\log\left(\dfrac{M}{M - Pr}\right)}{\log(1 + r)}$$ Use the time-to-pay-off formula.

$$n = \frac{\log\left(\dfrac{150}{150 - (1200)(0.0125)}\right)}{\log(1 + 0.0125)}$$ $P = 1200$; $r = 0.15 \div 12 = 0.0125$; $M = 150$

$$n = 8.48$$ To the nearest hundredth

Use the following keystrokes: ⊏(⊏LOG⊐ ⊏(150 ⊏÷⊐ ⊏(150 ⊏−⊐ 1200 ⊏×⊐
0.0125 ⊐) ⊐) ⊐) ⊏÷⊐ ⊏(⊏LOG⊐ ⊏(1.0125 ⊐) ⊐) ⊏ENTER⊐

It takes 9 months to pay off the balance. In the last month he owes less than $150.

EXAMPLE 2 Trevor's father just received an unexpected raise. As a result, he may be able to make a larger monthly payment to MasterCard.

QUESTION How long will it take him to pay off his bill if he makes monthly payments of $200 instead of $150?

SOLUTION

$$n = \frac{\log\left(\frac{M}{M - Pr}\right)}{\log(1 + r)}$$

$$n = \frac{\log\left(\frac{200}{200 - (1200)(0.0125)}\right)}{\log(1 + 0.0125)} \qquad P = 1200; \ r = 0.15 \div 12 = 0.0125; \ M = 200$$

$$n = 6.28$$

It takes 7 months to pay off the balance if he makes a monthly payment of $200 instead of $150.

--- **SKILL 2** ---

You may wish to pay off a credit card balance before a certain event occurs or before you need to make another purchase. For example, you may want to pay off a balance before you begin college. In this case you will want to know how much you have to pay each month to bring the balance to zero in a specified period of time.

Since the process of paying off a credit card balance is similar to making monthly payments on an installment bank loan, you can apply the monthly payment formula that you studied in Chapter 5. To use the formula for this purpose, let P represent the current balance.

> ### Monthly Payment Formula for Paying Off Balance
>
> $$M = \frac{Pr(1 + r)^n}{(1 + r)^n - 1}$$
>
> where P = current balance
> r = monthly interest rate
> n = number of payment periods
> M = monthly payment

EXAMPLE 3 Trevor's cousin Adam will be going to college in 2 years. Trevor's uncle would like to pay off his VISA card balance before he begins making tuition payments.

QUESTION If his current balance is $4200 and his card carries an annual interest rate of 21%, how large must his monthly payment be if the balance is to be paid off in 2 years?

SOLUTION

$$M = \frac{Pr(1 + r)^n}{(1 + r)^n - 1}$$

$$M = \frac{4200(0.0175)(1 + 0.0175)^{24}}{(1 + 0.0175)^{24} - 1}$$
$P = 4200;\ r = (0.21 \div 12) = 0.0175;$
$n = 2 \cdot 12 = 24$

$$M = 215.82$$

He must make monthly payments of $215.82 to pay off the balance in 2 years.

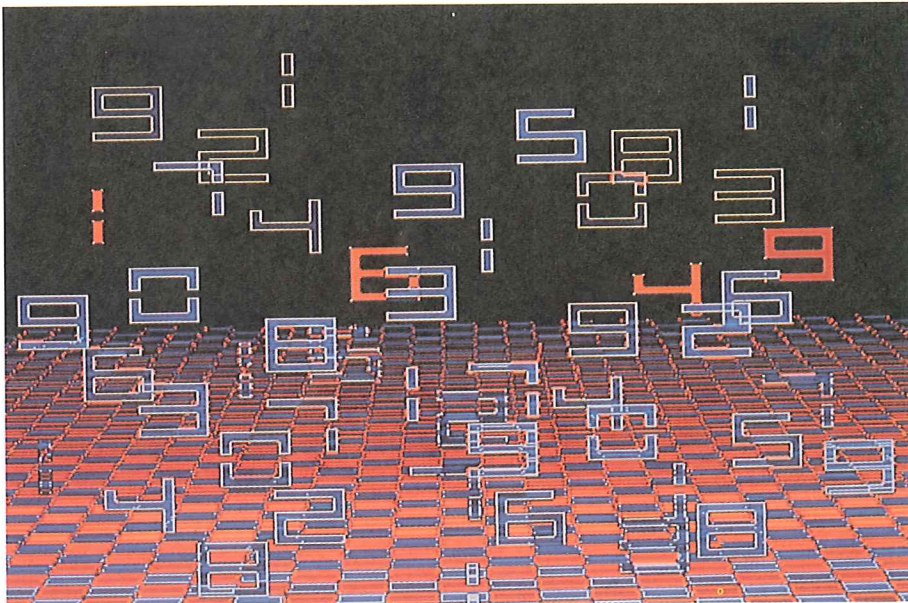## TRY YOUR SKILLS

Evaluate to the nearest hundredth.

1. $\log 1000 \div \log 17$
2. $\log 129 \div \log 1.02$
3. $\log (1200 \div 1100) \div \log 1.75$
4. $\log (180 \div 165) \div \log 1.01$

How long will it take to pay off each credit card balance with the given monthly payment at the given APR? Express your answer as a whole number of months.
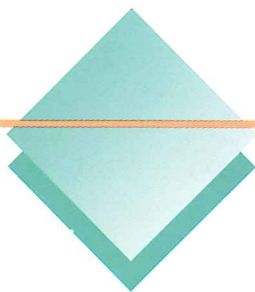
5. $1500; $170; 12%
6. $3500; $200; 14%
7. $1850; $225; 16%
8. $2000; $210; 15%

Determine the monthly payment that must be made to reduce each MasterCard balance to zero in the specified period of time. Assume that there is an annual finance charge of 15.9%.

9. $28,000 in $2\frac{1}{2}$ years
10. $16,000 in 25 months

1. If you determine that it will take 7.2 months to pay off a credit balance, why would you round your answer to 8 months instead of 7 months?

2. Why is the process of paying off a credit card balance similar to making monthly payments on an installment bank loan?

Evaluate to the nearest hundredth.

3. $\log 1280 \div \log 25$

4. $\log 277 \div \log 1.15$

5. $\log (1600 \div 1450) \div \log 1.22$

6. $\log (210 \div 195) \div \log 1.0167$

How long will it take to pay off each credit card balance with the given monthly payment at the given APR? Express your answer as a whole number of months.

7. $1500; $110; 12%   8. $170; $15; 12%   9. $1350; $85; 10.9%

10. $180; $36; 10.9%   11. $2750; $180; 16.9%   12. $325; $15; 16.9%

13. $3000; $225; 18%   14. $400; $28; 18%   15. $4500; $220; 12%

16. $525; $42; 12%   17. $5100; $325; 21%   18. $610; $45; 21%

Determine the monthly payment that must be made to reduce each VISA balance to zero in the specified period of time at the given APR.

19. $1900; $1\frac{1}{2}$ years; 15%

20. $2350; 22 months; 16.9%

21. $775; 16 months; 17.5%

22. $895; 15 months; 18%

23. $1675; $3\frac{1}{2}$ years; 9%

24. $1958; $2\frac{1}{2}$ years; 16%

MIXED REVIEW

1. How much money will you save by prepaying a loan of $34,000 at 12.5% for 10 years for which you have been making monthly payments of $497.68 if you prepay it at the end of 7 years?

2. Determine the effective rate of interest to the nearest hundredth of a percent if the APR is 14%.

3. Bevilaqua earns $566.66 in gross income each week. How much will be deducted for Social Security and Medicare?

4. If Marci can afford a monthly payment of $360, how much can she afford to borrow at a yearly interest rate of 9.5% for 5 years?

5. Use the Rule of 72 to determine how long it will take your $5500 savings account to double in value if it is growing at a rate of 3%.

6. If you buy living room furniture for $4100 and finance it at 16% over 3 years, how much will you save by making a down payment of 20% instead of 10%?

7. Suppose that the equation for the cost function of a small business is $c = 1.9n + 118.50$ and the equation for the revenue function is $r = 5n$. Use a graphing calculator to find the break-even point.