

AMORTIZATION SCHEDULES

When Maria receives her car loan, her monthly payment will be identical each month. However, the portion of that payment that is applied as interest changes each month. Banks or other lending institutions provide borrowers with schedules, called **amortization schedules**, that list the interest portion of monthly loan payments. These amortization schedules also list the payment number, the monthly payment, the reduction in unpaid balance (note reduction), and the remaining unpaid balance for each month.

The **payment number** corresponds to the number of months that have elapsed since the money was borrowed. The **interest due** is determined by multiplying the monthly interest rate by the previous month's unpaid balance. The **note reduction** is found by subtracting the interest due from the monthly payment and shows how much of the monthly payment actually goes to paying off the loan. The **unpaid balance** is calculated by subtracting the note reduction from the previous month's unpaid balance and represents how much of the original loan is still unpaid.

PREPAYMENT

Most financial institutions will allow a borrower to prepay the balance of an installment loan. This right of prepayment (paying off the contract before it comes due) saves the borrower interest charges. However, the amount saved by prepaying a loan varies according to each loan agreement. Most loan contracts require the borrower to pay more interest in the early months of the loan period when the lender's risks are greater and less interest near the maturity date when the lender's risks are less.

Many financial institutions use a standard formula to determine the remaining portion of a loan at the time the borrower chooses to prepay it. The formula is based upon the monthly payment the borrower has been making, the monthly interest rate, and the number of remaining payment periods.



RULE OF 78

Some stores apply the **Rule of 78**, instead of the prepayment formula used by financial institutions, to buyers who purchase merchandise on credit for 1 year and wish to prepay. Since the sum of the numbers from 1 to 12 is 78, the buyer

pays a portion of the yearly interest equal to $\frac{12}{78}$ in month 1, $\frac{11}{78}$ in month 2, $\frac{10}{78}$ in month 3, and so on, until $\frac{1}{78}$ is paid in month 12. The largest amount of interest is paid in month 1, and the smallest amount of interest is paid in month 12. By the end of the year, all the interest is paid.

In general, prepaying a loan saves the borrower money. However, some lending institutions charge a **prepayment penalty** to offset a portion of the lost revenues and additional clerical costs that they incur when a borrower prepays a loan. A borrower who is considering prepaying a loan must check the prepayment stipulations stated in the loan contract to determine the costs, if any, of prepayment. This is one of the many reasons that it is very important that the borrower understand the terms of a credit agreement before signing it.

Ask Yourself

1. What does an amortization schedule show?
2. Why do most loan contracts require the borrower to pay more interest in the early months of the loan period?
3. Do you think that you will save money if you prepay a loan?
4. Do you think that it is fair for a bank to charge a prepayment penalty?

ALGEBRA REVIEW

1. If the annual interest rate is 15%, what is the monthly interest rate?
2. Express your answer to Exercise 1 as a decimal.

Evaluate. Round to the nearest hundredth.

3. $x = 9 + 8 + 7 + \dots + 2$
4. $x = 15 + 14 + 13 + \dots + 5$
5. $x = \frac{3}{78} + \frac{2}{78}$
6. $x = \frac{7}{29} + \frac{6}{29} + \frac{5}{29}$
7. $x = 3 + 2^4$
8. $x = 3 + 2^{-4}$
9. $x = 1 + (3 + 2)^{-4}$
10. $x = 1 + (3 + 2)^4$
11. $x = 4.5[100 - (6 + 1)^{-12}]$
12. $x = 3.9[98 - (2.30122)^{-3}]$
13. $x = \frac{400[1 - (1 + 0.015)^{-10}]}{0.015}$
14. $x = \frac{325[1 - (1 + 0.0125)^{-24}]}{0.0125}$

SHARPEN YOUR SKILLS

SKILL 1

EXAMPLE 1 Maria is considering a \$4500 loan for 2 years at 9%. Her monthly payment is \$205.58.

QUESTION What are the interest due, note reduction, and unpaid balance for the first 2 months?

SOLUTION

To determine I_1 , the interest due at the end of month 1, Maria multiplied the loan amount L by the monthly interest rate r :

$$I_1 = rL \quad 0.0075 \cdot 4500 = \$33.75 \quad r = 0.09 \div 12 = 0.0075; L = 4500$$

To find R_1 , the note reduction in month 1, she subtracted I_1 , the interest due at the end of month 1, from the payment P .

$$R_1 = P - I_1 \quad 205.58 - 33.75 = \$171.83 \quad P = 205.58; I_1 = 33.75$$

To calculate B_1 , the unpaid balance at the end of month 1, she subtracted the note reduction R_1 from the original loan amount L .

$$B_1 = L - R_1 \quad 4500 - 171.83 = \$4328.17 \quad L = 4500; R_1 = 171.83$$

To determine I_2 , the interest due at the end of month 2, she multiplied B_1 , the unpaid balance from month 1, by the monthly interest rate r :

$$I_2 = rB_1 \quad 0.0075 \cdot 4328.17 = \$32.46 \quad r = 0.09 \div 12 = 0.0075; B_1 = 4328.17$$

To find the note reduction R_2 in month 2, she subtracted the interest I_2 due from the payment P .

$$R_2 = P - I_2 \quad 205.58 - 32.46 = \$173.12 \quad P = 205.58; I_2 = 32.46$$

To calculate the new unpaid balance B_2 , she subtracted the note reduction R_2 from the unpaid balance from month 1.

$$B_2 = B_1 - R_2 \quad 4328.17 - 173.12 = 4155.05 \quad B_1 = 4328.17; R_2 = 173.12$$

Maria was able to see a pattern in her calculations and to use the pattern to predict the following formula for the unpaid balance after 3 months.

$$B_3 = B_2 - R_3 \quad \text{where } R_3 = P - I_3 \quad \text{and } I_3 = rB_2$$

Maria realized that the pattern she observed could be extended to cover more than just the first 3 months of the loan. The pattern could be used to create an amortization schedule as long as she wanted.



AMORTIZATION SCHEDULE FORMULAS

$$\begin{array}{lll}
 I_1 = rL & I_2 = rB_1 & \text{where } L = \text{loan amount} \\
 R_1 = P - I_1 & R_2 = P - I_2 & r = \text{monthly interest rate} \\
 B_1 = L - R_1 & B_2 = B_1 - R_2 & P = \text{payment amount} \\
 & & I_1 = \text{interest due at end of} \\
 & & \quad \text{month 1} \\
 & & R_1 = \text{loan reduction at end} \\
 & & \quad \text{of month 1} \\
 & & B_1 = \text{balance at end} \\
 & & \quad \text{of month 1}
 \end{array}$$

EXAMPLE 2 Maria wants to know what her loan amortization schedule will look like for the first year of her loan for \$4500 for 2 years at 9%.

QUESTION What are her interest due, note reduction, and unpaid balance for the first 12 months?

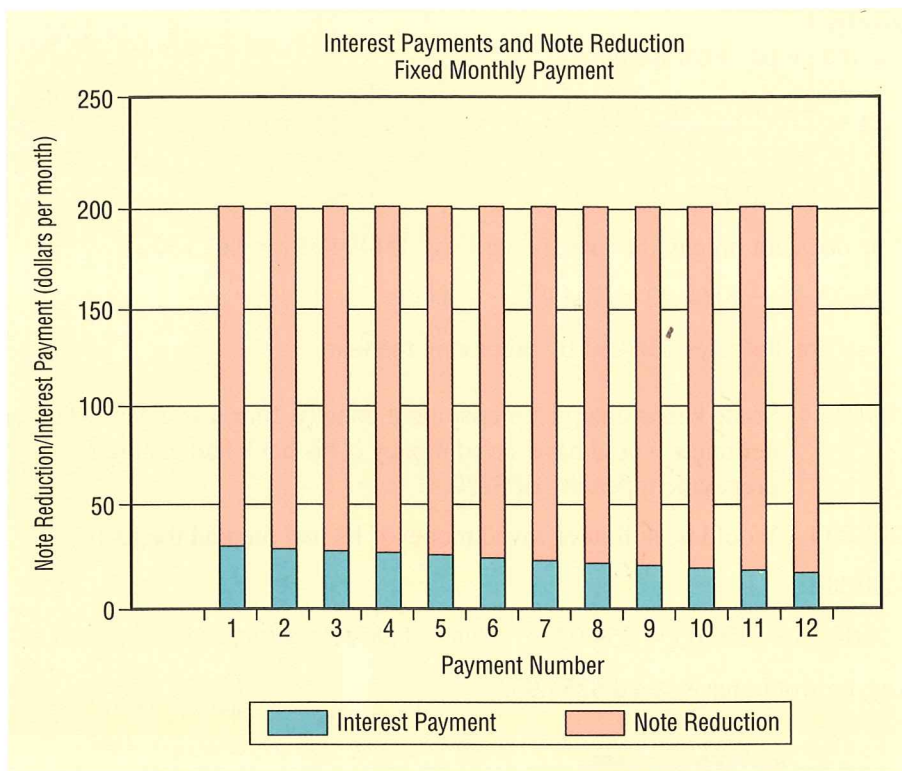


SOLUTION

Maria used her spreadsheet program to create an amortization schedule that shows the payment number, payment amount, interest due, note reduction, and unpaid balance for months 1–12. The rate $r = 0.09 \div 12 = 0.0075$.

	A	B	C	D	E
1	Payment	Payment	Interest	Note	Unpaid
2	Number	Amount	Due	Reduction	Balance
3		<code>@ROUND(E3*0.0075,2)</code>	<code>+B4-C4</code>	<code>+E3-D4</code>	4500.00
4	1	205.58	33.75	171.83	4328.17
5	2	205.58	32.46	173.12	4155.05
6	3	205.58	31.16	174.42	3980.63
7	4	205.58	29.85	175.73	3804.91
8	5	205.58	28.54	177.04	3627.86
9	6	205.58	27.21	178.37	3449.49
10	7	205.58	25.87	179.71	3269.78
11	8	205.58	24.52	181.06	3088.72
12	9	205.58	23.17	182.41	2906.31
13	10	205.58	21.80	183.78	2722.53
14	11	205.58	20.42	185.16	2537.37
15	12	205.58	19.03	186.55	2350.82

Next she used the graph function on her computer spreadsheet program to graph the portion of each monthly payment that is interest.



Note that the interest portion of the loan decreases each month even though the monthly payment remains constant.

SKILL 2

Maria would like to know how much she will save if she prepays her loan. She can use the prepayment formula to determine the remaining payment if she prepays it at any given time. She can then compare this amount with the amount that she would have paid if she did not prepay the loan.

PREPAYMENT FORMULA

$$A = \frac{M[1 - (1 + r)^{-q}]}{r}$$

where M = monthly payment
 r = monthly interest rate
 q = number of remaining payment periods
 A = prepayment amount

EXAMPLE 3 Maria's uncle decides to prepay his car loan. He has been making payments of \$444.89 per month on a 5-year loan of \$20,000 at 12%.

QUESTION How much does he owe if he prepays the loan 15 months early? How much money will he save?



SOLUTION

Use the prepayment formula.

$$A = \frac{444.89[1 - (1 + 0.01)^{-15}]}{0.01}$$

$$A = \$6168.42$$

$$\begin{aligned} M &= 444.89 \\ r &= 0.12 \div 12 = 0.01 \\ q &= 15 \end{aligned}$$

If he does not prepay the loan, he will pay $15(444.89) = 6673.35$.

$$6673.35 - 6168.42 = 504.93 \quad \text{Subtract.}$$

Therefore, he saves \$504.93 by prepaying the loan.

EXAMPLE 4 Some banks charge a prepayment penalty. Maria wonders whether her uncle would have saved money if his bank had charged a prepayment penalty of \$250.

QUESTION Would he still have saved money if he had prepaid the loan?

SOLUTION

$$504.93 - 250.00 = 254.93 \quad \text{Subtract the penalty from the original savings.}$$

Yes, he would have saved \$254.93.

SKILL 3



RULE OF 78

The buyer pays a portion of the yearly interest equal to

$$\frac{12}{78} \text{ in month 1}$$

$$\frac{11}{78} \text{ in month 2}$$

$$\frac{10}{78} \text{ in month 3}$$

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$$\frac{1}{78} \text{ in month 12}$$

EXAMPLE 5 Maria's mother purchased a microwave oven for \$360, putting \$60 down. She signed a contract stating that the balance of \$300 was to be paid in 12 equal installments with an interest rate of 21% per year.

QUESTION If she decides to prepay her contract at the time of her ninth payment and the store applies the Rule of 78, how much interest will she save?

SOLUTION

Find the yearly interest.

$$0.21 \cdot 300 = 63 \quad \text{The yearly interest rate is 21\%.}$$

After 9 months, Maria's mother has three remaining payments. By prepaying the loan she will not have to pay the portion of yearly interest due in the last three months. Apply the Rule of 78.

$$\frac{3}{78} + \frac{2}{78} + \frac{1}{78} = \frac{6}{78}$$
$$\frac{6}{78} \cdot 63 = 4.85 \quad \text{The yearly interest is \$63.}$$

Therefore she saves \$4.85.

TRY YOUR SKILLS

Create an amortization schedule for the first 3 months of a \$2000 loan at 6% for 4 years for which the monthly payment is \$46.97.

	Payment Number	Payment Amount	Interest Due	Note Reduction	Unpaid Balance
1.	1	\$46.97	\$10.00	\$36.97	
2.	2	46.97			
3.	3				

Determine the prepayment savings on a bank loan of \$25,000 at 12% for 30 years for which you have been paying \$257.15 each month under each condition. Assume that there are no prepayment penalties.

4. It is prepaid 20 months early. Use the prepayment formula.

$$A = \frac{M[1 - (1 + r)^{-q}]}{r} \quad \text{Substitute } M = 257.15, \\ r = 0.12 \div 12 = 0.01, \text{ and } q = 20.$$

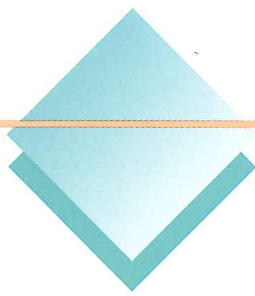
$$A = \frac{257.15[1 - (1 + 0.01)^{-20}]}{0.01}$$

5. It is prepaid 3 years early.

A \$600.00 television purchase is financed at 18% per year with no down payment at a store that applies the Rule of 78. It must be paid in 12 equal installments. Determine the prepayment savings under each condition.

6. It is prepaid after 4 months.
7. It is prepaid after 7 months.





EXERCISE YOUR SKILLS

KEY TERMS

amortization
 schedule
 interest due
 note reduction
 payment number
 prepayment penalty
 Rule of 78
 unpaid balance

1. If your bank charges a \$200 prepayment penalty, under what condition will you consider prepaying your loan?
2. Why does the interest portion of a monthly payment decrease each month?
3. What is an amortization schedule?

Gregory borrows \$9500 from a bank to purchase a car. He finances it at 8% for 4 years, and his monthly payment is \$231.92. Use a spreadsheet program to create an amortization schedule that shows the payment number, payment amount, interest due, note reduction, and unpaid balance for the first year.

	Payment Number	Payment Amount	Interest Due	Note Reduction	Unpaid Balance
4.					
5.					
6.					
7.					
8.					
9.					
10.					
11.					
12.					
13.					
14.					
15.					

Determine the prepayment savings on a bank loan of \$21,400 at 11% for 25 years for which you have been paying \$209.74 each month under each condition. Assume that there are no prepayment penalties.

16. It is prepaid 37 months early. 17. It is prepaid 5 years early.

Determine the prepayment savings on a bank loan of \$100,000 at 12.5% for 15 years for which you have been paying \$1232.52 each month under each condition. Assume that there is a prepayment penalty of \$500.

18. It is prepaid 42 months early. 19. It is prepaid 6 years early.

Use the Rule of 78 to determine how much interest will be saved by prepaying a one-year loan of \$1500 at 10% after each time period.

20. 4 months 21. 6 months 22. 10 months

MIXED REVIEW

- Determine which of the following loans has a greater total cost:
 - \$10,000 at 12% for 5 years
 - \$11,000 at 11% for 3 years
- During the first week of April, Brielle worked 34 hours and received \$44.95 in tips. If her hourly rate of pay is \$5.75, how much did she earn that week?
- Carmen has \$8500 in a CD that pays 6% interest, compounded semiannually. How much is her CD worth at the end of 2 years?
- Angela earns \$125 in gross income each week and is single and claims one exemption. How much will her employer withhold for income tax?
- Rachel receives a commission of 4% on sales up to \$10,000 and 6% on all sales over \$10,000. How much has she earned in commission if her sales are \$18,250?

James Wilcox is a farmer with 160 acres on which to plant. From crop x he can earn \$200 per acre; from crop y he can earn \$150 per acre. To meet demands, he must plant at least 40 acres of crop x and 60 acres of crop y . Soil conditions will permit him to plant no more than 100 acres of crop x .



- Write the objective function for his total income.
- Write inequalities that express each constraint.
- Graph the inequalities.
- Find the corners of the region in Exercise 8.
- What is the number of acres of each crop that James Wilcox should plant to maximize his income?



THE LURE OF CREDIT TERMS: THE MERCHANT PROFITS



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aria discovered that there are methods to reduce the cost of credit. Selecting a shorter amortization period, a lower interest rate, and a larger down payment enables the borrower to reduce the total payments on a loan. In addition, prepaying a loan can often save the borrower money. Many car ads mention terms that include cash-back opportunities and

significantly lower interest rates than financial institutions are offering.

Maria wonders whether she can save money on the purchase of her car by taking advantage of some of these terms. Patrick warns that merchants can increase their profits by enticing customers to choose credit terms that are favorable to the merchants.

OBJECTIVES: *In this lesson, we will help Maria to:*

- *Learn how merchants generate credit income and attract customers with special credit plans.*
- *Calculate the total financed price with lower-than-market-rate interest and rebate plans.*
- *Determine the merchants' profits with lower-than-market-rate interest and rebate plans.*
- *Observe the drawbacks associated with rent-to-own plans.*