4-4 BREAK-EVEN POINT

Evelyn and her friends, like all businesspeople, have no guarantees. When they began their business venture, they did not know whether they would have many, few, or even no customers. These students were determined but also a little anxious. They wanted to succeed. In business the primary indicator of success is making a profit. As we have seen, this will occur when revenues are greater than costs.

Usually, the more efficient a business is, the greater its profits will be. A business can increase its profits by offering consumers better products, lower prices, and better services than competitors. Thus consumers benefit from the efforts of businesses to make a profit.

Profits are important to a business, not because businesspeople are greedy, but because extra money is needed to investigate new business opportunities, to set aside reserves for times when the company does not make a profit, and to provide for emergencies.

Profit, then, is the incentive or reward for improved production and services. It was the desire for profit that led Evelyn and the others to decide on the T-shirt business rather than word processing, and it was the profit motive that helped them determine the appropriate quantities to buy and sell. Finally, it was their interest in profits that led these students to ask Hari to help them understand the finances of the businesses.

The first thing Hari taught the other students was that they should be aware of the break-even point that separates losses from profits.

OBJECTIVES: *In this lesson, we will help Hari to:*

- *Graph the cost and revenue functions.*
- *Determine the break-even point algebraically.*
- *Determine the break-even point using the graphing calculator.*

ALGEBRA *REVIEW*

Write in slope-intercept form, and find the slope.

1. $x + y = 10$
2. $y = 2x - 3$
3. $3y = 2x + 6$
4. $2x - 4y = 20$

Graph each equation.

5. $y = 2x - 5$
6. $2x - 3y = 12$

Find the point of intersection of each pair of equations.

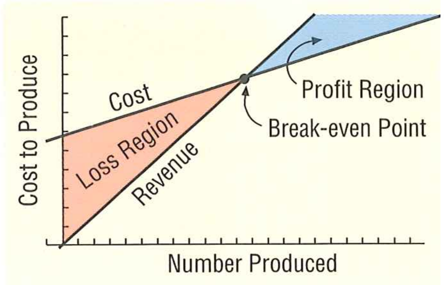
7. $x + y = 8$
   $x - y = 12$
8. $y = 2x$
   $2x + 2y = 18$

Use a graphing calculator to approximate the points of intersection to the nearest hundreth.

9. $2x + 5y = 17$
   $3x - y = 21$
10. $4y - 3.7x = 16$
    $3.4x + 2y = 34.6$

BREAK-EVEN POINT

We have seen that establishing prices is a most important step for any business. Prices and the number of products sold determine a company's revenue. If a business's total cost is greater than its revenue, then the business incurs a *loss.* If the revenue is greater than the total cost, then the business makes a *profit.* The point at which cost and revenue are equal is called the *break-even point.* This point is given numerically by the number of products sold at a given price. It is also useful to illustrate the break-even point by graphing the line for cost and the line for revenue. The break-even point is where these two lines intersect.



The region between the lines and above the break-even point is the **profit region.** The region between the lines and below the break-even point is the **loss region.**

Ask Yourself

1. Why are profits important to a business?
2. What is the break-even point?
3. What factors determine the break-even point?

SHARPEN YOUR SKILLS

SKILL 1

EXAMPLE 1 Hari wants to find the break-even point for the T-shirt business when the fixed cost is \$282, the unit cost is \$6.77, and the selling price is \$19.

QUESTION How can a graph be used to show the break-even point for Hari's T-shirt business?

SOLUTION

From Lesson 4-3, Example 1, the equation for the cost function is

$$c = 6.77n + 282 \qquad \text{n = the number of T-shirts made}$$

and the equation for the revenue function is

$$r = 19n \qquad \text{n = the number of T-shirts sold}$$

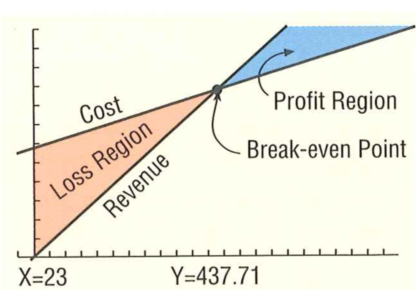
Since c and r both refer to amounts of money, you can use the same axis to graph them. Since the amount of money depends on the number of T-shirts, use y for c and r; and x for n, the number of T-shirts. The equations are

$$y = 6.77x + 282$$
$$y = 19x$$

Graph these equations on a graphing calculator with the following range values.

Xmin:	−2	Ymin:	−75
Xmax:	45.5	Ymax:	600
Xscl:	2	Yscl:	50

The break-even point is the point where the graphs intersect. The region between the lines and above the break-even point is the *profit region.* In this region the revenue is greater than the cost. The region between the lines and below the break-even point is the *loss region.* In this region the cost is greater than the revenue. The break-even point is the point where the cost and the revenue are *equal.* Using the trace function the break-even point appears to be (23, 437.71).



SKILL 2

EXAMPLE 2 Hari wonders if there is a way to determine the break-even point algebraically.

QUESTION How can Hari solve the equations of Example 1 algebraically to find the break-even point?

SOLUTION

A system of two linear equations can be solved by the addition and subtraction method or by substitution. In this case, substitution appears to be the more direct method.

$y = 6.77x + 282$	(1) Cost equation
$y = 19x$	(2) Revenue equation
$19x = 6.77x + 282$	Substituting for y in (1)
$12.23x = 282$	Subtract $6.77x$ from both sides.
$x = 23.06$	To the nearest hundredth
$y = 6.77(23.06) + 282$	Substitute for x in (1).
$y = 438.1$	To the nearest tenth.
$y = 19(23.06)$	Substitute for x in (2).
$y = 438.1$	To the nearest tenth.

We need to interpret the meaning of the point (23.06, 438.1). Since only whole T-shirts can be made, x values need to be whole numbers. Since making 23 T-shirts would result in a loss, although small, the students need to make 24 T-shirts to break even. The break-even point is 24 T-shirts.

If $x = 23$	If $x = 24$
Cost: $6.77(23) + 282 = 437.71$	Cost: $6.77(24) + 282 = 444.48$
Revenue: $19(23) = 437$	Revenue: $19(24) = 456$
Loss: $437 - 437.71 = -0.71$	Profit: $456 - 444.48 = 11.52$

TRY YOUR SKILLS

Students selling bumper stickers at $2.00 each have the following costs.

Fixed costs: Labor, 50 hours at $4.20 per hour
Advertising, energy, and transportation, $32.50
Variable costs: $1.04 per sticker

1. What is the cost function?
2. What is the revenue function?
3. Using algebraic methods, solve the cost function and revenue function simultaneously to find the break-even point.
4. Graph the equations to show the regions of profit and loss.

EXERCISE YOUR SKILLS

1. The break-even point is the intersection of the graphs of what two functions?
2. Why is the break-even point important to a businessperson?

Students selling pennants at $2.00 each have the following costs.

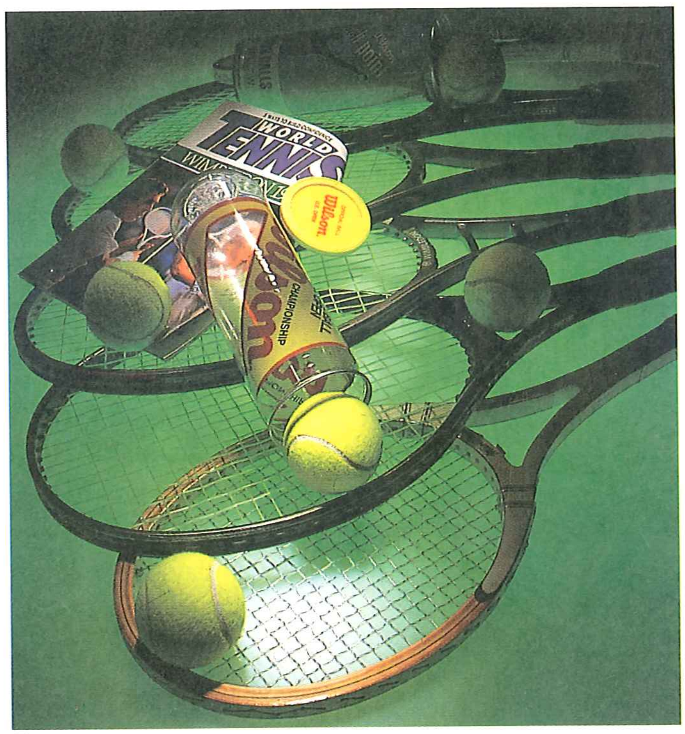
Fixed costs: Labor, 40 hours at $4.50 per hour
Advertising, energy, and transportation, $32.50
Variable costs: Pennants, $1.00 each
Paints, $0.20 per pennant

3. What is the cost function?
4. What is the revenue function?
5. Find the break-even point algebraically.

Students selling tennis balls at $4.00 per can have the following costs.

Fixed costs: Labor, 20 hours at $4.00 per hour
Advertising, energy, and transportation, $32.50
Variable costs: Balls, $2.75 per can

6. What is the cost function?
7. What is the revenue function?
8. Graph the cost function and revenue function.
9. Find the break-even point using calculator or computer features.



Students selling book bags at $9.00 each have the following costs.

Fixed costs: Labor, 35 hours at $4.50 per hour
Advertising, energy, and transportation, $32.50
Variable costs: Bags, $4.00 each

10. What is the cost function?
11. What is the revenue function?
12. Find the break-even point using any method.
13. Graph the cost function and revenue function.
14. What is the profit function?
15. Find the point at which the business suffers a loss of $65.
16. Find the point at which the business gains a profit of $75.

KEY TERMS

break-even point
loss region
profit region

Students selling personalized towels at $5.00 each have the following costs.

Fixed costs: Labor, 60 hours at $4.00 per hour
Advertising, energy, and transportation, $32.50

Variable costs: Towels, $2.00 each
Letters, $0.20 per towel
Cardboard boxes, $0.10 each

17. What is the cost function?

18. What is the revenue function?

19. Graph the cost and revenue functions.

20. Find the break-even point.

21. What is the profit function?

22. Find the point at which the business suffers a loss of $78.10.

23. Find the point at which the business gains a profit of $194.60.

MIXED REVIEW

1. Veronica earns a commission of $72 on a sale of $600. What is her percent of commission?

2. Find the value to which $500 grows when it is invested for 5 years at 6.5% interest compounded annually.

In Exercises 3–6, suppose that the Federal Reserve requires that 20% of an initial deposit of $5,000 be held in reserve but that the balance may be lent out by the bank.

3. Show the first two levels of extra money that is generated.

4. What is the total amount of new money that is potentially created by the multiplier effect?

5. What is the multiplier for the case in which the reserve requirement is 20%?

6. If the reserve requirement were raised to 25%, what would be the value of the multiplier?

Find the cost of producing the given amounts of the following items. In both cases, the cost of advertising, energy, and transportation is $47.50.

7. Five students are making personalized school calendars. They expect to prepare 25 of the calendars per week. What will be the cost of production if the time required is 20 hours at $4.75 per hour and the cost of materials is $1.20 per calendar?

8. Two students plan to produce coin dispensers for highway tolls. How much will it cost to produce 30 dispensers if the time for the labor is 25 hours at $5.50 per hour and the cost of materials is $0.83 per dispenser?