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Study Guide

The Remainder and Factor Theorems

The Remainder Theorem	If a polynomial $P(x)$ is divided by $x - r$, the remainder is a constant $P(r)$, and $P(x) = (x - r) \cdot Q(x) + P(r)$ where $Q(x)$ is a polynomial with degree one less than the degree of $P(x)$.

Example 1 **Divide $x^4 - 5x^2 - 17x - 12$ by $x + 3$.**

$$
\begin{array}{r}
x^3 - 3x^2 + 4x - 29 \\
x + 3\overline{)\,x^4 + 0x^3 - 5x^2 - 17x - 12} \\
\underline{x^4 + 3x^3} \\
-3x^3 - 5x^2 \\
\underline{-3x^3 - 9x^2} \\
4x^2 - 17x \\
\underline{4x^2 + 12x} \\
-29x - 12 \\
\underline{-29x - 87} \\
75 \leftarrow \textit{remainder}
\end{array}
$$

Find the value of r in this division.

$x - r = x + 3$

$-r = 3$

$r = -3$

According to the Remainder Theorem, $P(r)$ or $P(-3)$ should equal 75.

Use the Remainder Theorem to check the remainder found by long division.

$$
\begin{aligned}
P(x) &= x^4 - 5x^2 - 17x - 12 \\
P(-3) &= (-3)^4 - 5(-3)^2 - 17(-3) - 12 \\
&= 81 - 45 + 51 - 12 \text{ or } 75
\end{aligned}
$$

The Factor Theorem is a special case of the Remainder Theorem and can be used to quickly test for factors of a polynomial.

The Factor Theorem	The binomial $x - r$ is a factor of the polynomial $P(x)$ if and only if $P(r) = 0$.

Example 2 **Use the Remainder Theorem to find the remainder when $2x^3 + 5x^2 - 14x - 8$ is divided by $x - 2$. State whether the binomial is a factor of the polynomial. Explain.**

Find $f(2)$ to see if $x - 2$ is a factor.

$$
\begin{aligned}
f(x) &= 2x^3 + 5x^2 - 14x - 8 \\
f(2) &= 2(2)^3 + 5(2)^2 - 14(2) - 8 \\
&= 16 + 20 - 28 - 8 \\
&= 0
\end{aligned}
$$

Since $f(2) = 0$, the remainder is 0. So the binomial $x - 2$ is a factor of the polynomial by the Factor Theorem.

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Practice

The Remainder and Factor Theorems

Divide using synthetic division.

1. $(3x^2 + 4x - 12) \div (x + 5)$

2. $(x^2 - 5x - 12) \div (x - 3)$

3. $(x^4 - 3x^2 + 12) \div (x + 1)$

4. $(2x^3 + 3x^2 - 8x + 3) \div (x + 3)$

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

5. $(2x^4 + 4x^3 - x^2 + 9) \div (x + 1)$

6. $(2x^3 - 3x^2 - 10x + 3) \div (x - 3)$

7. $(3t^3 - 10t^2 + t - 5) \div (t - 4)$

8. $(10x^3 - 11x^2 - 47x + 30) \div (x + 2)$

9. $(x^4 + 5x^3 - 14x^2) \div (x - 2)$

10. $(2x^4 + 14x^3 - 2x^2 - 14x) \div (x + 7)$

11. $(y^3 + y^2 - 10) \div (y + 3)$

12. $(n^4 - n^3 - 10n^2 + 4n + 24) \div (n + 2)$

13. Use synthetic division to find all the factors of $x^3 + 6x^2 - 9x - 54$ if one of the factors is $x - 3$.

14. ***Manufacturing*** A cylindrical chemical storage tank must have a height 4 meters greater than the radius of the top of the tank. Determine the radius of the top and the height of the tank if the tank must have a volume of 15.71 cubic meters.