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Study Guide

Quadratic Equations

A quadratic equation is a polynomial equation with a degree of 2. Solving quadratic equations by graphing usually does not yield exact answers. Also, some quadratic expressions are not factorable. However, solutions can always be obtained by **completing the square.**

Example 1 **Solve $x^2 - 12x + 7 = 0$ by completing the square.**

$x^2 - 12x + 7 = 0$	
$x^2 - 12x = -7$	*Subtract 7 from each side.*
$x^2 - 12x + 36 = -7 + 36$	*Complete the square by adding* $\left[\frac{1}{2}(-12)\right]^2$, *or 36, to each side.*
$(x - 6)^2 = 29$	*Factor the perfect square trinomial.*
$x - 6 = \pm\sqrt{29}$	*Take the square root of each side.*
$x = 6 \pm \sqrt{29}$	*Add 6 to each side.*

The roots of the equation are $6 \pm \sqrt{29}$.

Completing the square can be used to develop a general formula for solving any quadratic equation of the form $ax^2 + bx + c = 0$. This formula is called the **Quadratic Formula** and can be used to find the roots of any quadratic equation.

Quadratic Formula	If $ax^2 + bx + c = 0$ with $a \neq 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In the Quadratic Formula, the radicand $b^2 - 4ac$ is called the **discriminant** of the equation. The discriminant tells the nature of the roots of a quadratic equation or the zeros of the related quadratic function.

Example 2 **Find the discriminant of $2x^2 - 3x = 7$ and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.**

Rewrite the equation using the standard form $ax^2 + bx + c = 0$.

$2x^2 - 3x - 7 = 0 \quad a = 2, b = -3, \text{ and } c = -7$

The value of the discriminant $b^2 - 4ac$ is $(-3)^2 - 4(2)(-7)$, or 65.

Since the value of the discriminant is greater than zero, there are two distinct real roots.

Now substitute the coefficients into the quadratic formula and solve.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{65}}{4}$$

The roots are $\frac{3 + \sqrt{65}}{4}$ and $\frac{3 - \sqrt{65}}{4}$.

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Practice

Quadratic Equations

Solve each equation by completing the square.

1. $x^2 - 5x - \frac{11}{4} = 0$

2. $-4x^2 - 11x = 7$

Find the discriminant of each equation and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

3. $x^2 + x - 6 = 0$

4. $4x^2 - 4x - 15 = 0$

5. $9x^2 - 12x + 4 = 0$

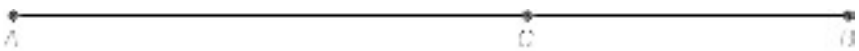
6. $3x^2 + 2x + 5 = 0$

Solve each equation.

7. $2x^2 + 5x - 12 = 0$

8. $5x^2 - 14x + 11 = 0$

9. ***Architecture*** The ancient Greek mathematicians thought that the most pleasing geometric forms, such as the ratio of the height to the width of a doorway, were created using the *golden section*. However, they were surprised to learn that the golden section is not a rational number. One way of expressing the golden section is by using a line segment. In the line segment shown, $\frac{AB}{AC} = \frac{AC}{CB}$. If $AC = 1$ unit, find the ratio $\frac{AB}{AC}$.



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Enrichment

Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let $z = x + y\boldsymbol{i}$. We denote the conjugate of z by $\bar{z}$. Thus, $\bar{z} = x - y\boldsymbol{i}$. We can define the absolute value of a complex number as follows.

$$|z| = |x + y\boldsymbol{i}| = \sqrt{x^2 + y^2}$$

There are many important relationships involving conjugates and absolute values of complex numbers.

Example **Show that $z^2 = z\bar{z}$ for any complex number z.**

Let $z = x + y\boldsymbol{i}$. Then,

$$\begin{aligned} z\bar{z} &= (x + y\boldsymbol{i})(x - y\boldsymbol{i}) \\ &= x^2 + y^2 \\ &= \left(\sqrt{x^2 + y^2}\right)^2 \\ &= |z|^2 \end{aligned}$$

Example **Show that $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse for any nonzero complex number z.**

We know that $|z|^2 = z\bar{z}$. If $z \neq 0$, then we have $z\left(\frac{\bar{z}}{|z|^2}\right) = 1$. Thus, $\frac{\bar{z}}{|z|^2}$ is the multiplicative inverse of z.

For each of the following complex numbers, find the absolute value and multiplicative inverse.

1. $2\boldsymbol{i}$

2. $-4 - 3\boldsymbol{i}$

3. $12 - 5\boldsymbol{i}$

4. $5 - 12\boldsymbol{i}$

5. $1 + \boldsymbol{i}$

6. $\sqrt{3} - \boldsymbol{i}$

7. $\frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3}\boldsymbol{i}$

8. $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\boldsymbol{i}$

9. $\frac{1}{2} - \frac{\sqrt{3}}{2}\boldsymbol{i}$