RULE OF 72

f you invest \$1000 at an annually compounded rate of 3% and make no withdrawals, your balance will double to \$2000 in about $23\frac{1}{2}$ years; the time for the \$2000 to redouble to \$4000 at a 3% rate is also about $23\frac{1}{2}$ years. The **Rule of 72** offers a fast and reasonably accurate way to letermine how long it will take for a sum of money to louble when it is compounded over several years. To find he time for an investment to double, divide 72 by the annual interest rate times 100. Since 10% • 100 = 10, just gnore the percent symbol. For example, if your \$5000 nvestment has an annual return of 10%, then in roughly 7 years (72 ÷ 10 = 7.2) your money will have doubled to \$10,000.

Another use for the Rule of 72 is to determine how long it will take for a currency to lose half its purchasing power. If the annual inflation rate stays at 6% for 12 years (72 ÷ 6 = 12), then the currency's purchasing power will drop by 50%.

The Rule of 72 is useful in calculating how long it will take you to save for a special long-term purpose, such as college expenses or retirement.

Ask Yourself

1. What is simple interest?
2. What is compound interest?
3. Do you think you will earn more interest on a savings account if the compounding is done quarterly rather than semiannually? Will the difference in the interest be very large?
4. What advantages do you give up to earn interest in a savings account rather than in some other kind of investment?

ALGEBRA REVIEW

The definition of an exponent is

$$a^m = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ factors}}$$

When multiplying like bases, add exponents.

$$a^m \cdot a^n = a^{m+n}$$

Simplify.

Example

$$(x^2 \cdot x^3) \cdot x \cdot x \cdot x$$
$$= (x^2 \cdot x^3) \cdot x^3$$
$$= x^5 \cdot x^3$$
$$= x^8$$

1. $x^1 \cdot x^3$
2. $2 \cdot 2^3$
3. $x^6 \cdot x^2$
4. $b^6 \cdot b^2$
5. $3y^4 \cdot 5y^4$
6. $(3y)^4\,(5)^4 y$
7. $(a \cdot a) \cdot a^4$
8. $p(1 + r)^n \cdot (1 + r)$

Simplify. Use the exponent key x^y of a calculator. Give answers to the nearest hundredth.

Example.

$$[(1.02)^4]\,[(1.02)(1.02)(1.02)]$$
$$= (1.02)^4\,(1.02)^3$$
$$= 1.02^7$$
$$= 1.15$$

9. $15{,}000(1.01)^3(1.01)$
10. $(1.035)^2(1.035)(1.035)$

SHARPEN YOUR SKILLS

SKILL 1

To find semiannual interest, multiply the principal p by the semiannual rate $r \div 2$ since $i = prt$ and $t = \frac{1}{2}$.

EXAMPLE 1 Nelson's parents have a certificate of deposit in the amount of \$10,000. It is held by a bank that pays 5% interest, compounded semiannually.

QUESTION How much will Nelson's parents have in this account after 2 years?

SOLUTION

Remember that the semiannual rate for 5% annual interest is $5\% \div 2$, or $2\frac{1}{2}\%$.

$$i = \text{principal} \cdot \text{semiannual rate}$$
$$i = 10{,}000 \cdot 0.025$$
$$i = 250$$

This interest is added to the principal of \$10,000. The new balance is \$10,250, so the next time that interest is paid, it will be computed on a balance of \$10,250. Each time, the interest is added to the balance before the next interest is calculated.

At the end of 2 years the family's balance sheet will look like this.

Time Period	Interest Earned	Principal Balance
Beginning	\$ 0.00	\$10,000.00
End of first half-year	250.00	10,250.00
End of first year	256.25	10,506.25
End of third half-year	262.66	10,768.91
End of second year	269.22	11,038.13

To obtain the interest earned in two years, use the formula

$$p + i = B$$

where p is the original principal, i is the interest and B is the balance at the end of two years.

$$p + i = B$$
$$i = B - p \quad \text{Subtract } p \text{ from both sides.}$$
$$i = 11{,}038.13 - 10{,}000 \quad B = 11{,}038.13 \text{ and } p = 10{,}000$$
$$i = 1038.13$$

The interest earned over two years is \$1038.13.

SKILL 2

Nelson wanted to know whether he could predict how much interest the family would have obtained if the CD had been held for more than two years. He decided to see whether there was a pattern to the numbers in the table. First he wrote the bank balance at the end of six months in the following form.

$$
\begin{aligned}
p + i &= 10{,}000 + 10{,}000 \bullet 0.025 \\
&= 10{,}000 \bullet 1 + 10{,}000 \bullet 0.025 && x = x \bullet 1 \\
&= 10{,}000 \bullet (1 + 0.025) && ab + ac = a(b + c)
\end{aligned}
$$

The balance at the end of six months was 10,000 • 1.025. Nelson had found the following pattern:

$$
\begin{aligned}
\text{New balance} &= \text{old balance} \bullet (1 + 0.025) \\
B_1 &= p(1.025)
\end{aligned}
$$

where p is the original principal and B_1 (read as "B sub one") is the total balance at the end of one six-month period.

Next Nelson noticed that at the end of two interest periods, the previous new balance became the new old balance. Thus at the end of two periods (1 year),

$$
\begin{aligned}
\text{New balance} &= \text{old balance} \bullet 1.025 && 1 + 0.025 = 1.025 \\
B_2 &= p(1.025) \bullet 1.025 \\
B_2 &= p(1.025)(1.025) \\
B_2 &= p(1.025)^2 && a \bullet a = a^2
\end{aligned}
$$

where B_2 is the account balance at the end of two six-month periods.

Nelson continued this pattern and found that at the end of three six-month periods the account balance would be

$$
\begin{aligned}
B_3 &= p(1.025)^2 \bullet (1.025) \\
&= p(1.025)^2 \bullet (1.025)^1 \\
&= p(1.025)^3 && a^2 \bullet a^1 = a^{2+1} = a^3
\end{aligned}
$$

From this pattern, Nelson was able to discover the following **compound interest formula** for the balance B in the account at any future date.

Compound Interest Formula

$B = p(1 + r)^n$ where B = balance
p = original principal
r = interest rate for the time period
n = total number of time periods

Nelson wondered how he might use the new formula that he had discovered.

EXAMPLE 2 Nelson decided to try the formula to find how much the CD would be worth at the end of five years.

QUESTION How much will a $10,000 CD at 5% be worth in five years?

SOLUTION

$B = p(1 + r)^n$ — Compound interest formula

$B = 10{,}000\ (1 + 0.025)^{10}$ — $r = 5\% \div 2$, or 2.5%; $n = 10$ six-month periods

$B = 12{,}800.85$ — Use your calculator.

The CD would be worth about $12,800 at the end of five years.

To enter the calculation in your graphing calculator use the exponent key [x^y], sometimes labeled [^] or [a^b]. The Enter key is sometimes labeled [EXE].

10000 [(] 1 [+] 0.025 [)] [x^y] 10 [ENTER]

SKILL 3

According to the Rule of 72, Nelson can find the speed at which his invested money will double by dividing 72 by 100 times the annual interest rate. Nelson would like to know whether he will be a millionaire by investing in CDs.

Rule of 72

$$\frac{72}{\text{Annual interest rate} \bullet 100} = \text{years to double}$$

EXAMPLE 3 Nelson invests $10,000 in a CD that pays 6% compounded quarterly.

QUESTION How long will it take his investment to double?

SOLUTION

Using the Rule of 72,

$72 \div 6 = 12$ — $6\% \bullet 100 = 6$

It will take 12 years to double the investment.

Nelson decided to check this result using the compound interest formula. If the interest is compounded quarterly, then the rate for each period is

$r = 0.06 \div 4 = 0.015$

If he invests the money for 12 years, then the number of periods is

$n = 12 \bullet 4 = 48$

Substitute in the compound interest formula.

$B = p(1 + r)^n$ — Compound interest formula

$B = 10{,}000(1 + 0.015)^{48}$ — Use a calculator.

$B = 20{,}435$ — To the nearest dollar

Since 20,435 is approximately 2 • 10,000, the result is confirmed; $10,000 has approximately doubled to $20,435.

EXAMPLE 4 Next, Nelson decided to continue the doubling to see how many 12-year periods he needed to reach $1,000,000.

QUESTION How long will it take his investment to be worth more than 1 million dollars?

SOLUTION

He noticed that in 12 years, the value of the investment had grown to about 2.04 times its earlier value (20,435 ÷ 10,000 = 2.0435). This was not exactly a double, but since he did not need an exact result, he believed that he could get a very good estimate by using 2 instead of 2.04. He made a table to keep track of the doublings.

Doublings	Years Passed	Balance
1	12	$ 20,435
2	24	40,870
3	36	81,740
4	48	163,480
5	60	326,960
6	72	653,920
7	84	1,307,840

It will take 7 doublings or 84 years to reach one million dollars. If Nelson is to become a millionaire through investments, he should either invest more money to begin with or find an investment that will pay a higher rate of return.

SKILL 4

EXAMPLE 5 Nelson has given up on the idea of becoming a millionaire quickly. Instead, he will invest $20,000 in an investment plan that he hopes will provide 10% interest, compounded quarterly.

QUESTION Nelson now wants to have $100,000 in his investment plan by the time he retires. How long will that take?

SOLUTION

Nelson substitutes all the values he can into the compound interest formula. The current value of his principal p is $20,000, the balance B that he hopes to have in the future is $100,000, and the quarterly rate of interest r that he hopes the investment will pay is 0.10 ÷ 4 = 0.025.

$B = p(1 + r)^n$ — Use the compound interest formula.

$100{,}000 = 20{,}000(1 + 0.025)^n$ — $B = 100{,}000$; $p = 20{,}000$; $r = 0.025$

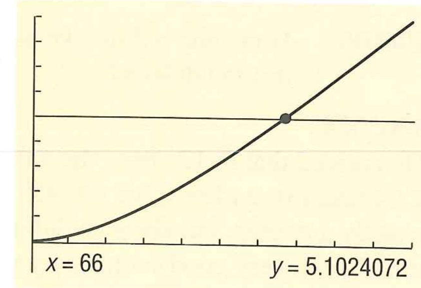
$5 = 1.025^n$ — Divide each side by 20,000.

Nelson is not sure how to solve this equation for n. He decides to use his graphing calculator. For each side of the above equation he writes a new equation, using the variable x for n.

$$y = 1.025^x$$
$$y = 5$$

He graphs them using these range values.

Xmin: 0
Xmax: 95
Xscl: 10
Ymin: 0
Ymax: 10
Yscl: 1



The horizontal line is the graph of $y = 5$ and the steep, rising curve (called an **exponential curve**) is the graph of $y = 1.025^x$. Nelson moves the trace cursor along the curve until he gets as close as he can to $y = 5$ without being less than 5. He finds that when $x = 66$, $y = 5.1024072$, so he substitutes 66 for n in the compound interest formula.

$B = 20{,}000(1.025)^{66}$
$B = 102{,}048$ To the nearest dollar

Nelson's \$20,000 investment will grow to over \$100,000 when $n = 66$. Since n represents the number of *quarters* (not years), Nelson needs to divide by 4 to find the number of years that it will take for him to retire.

Number of years $= 66 \div 4 = 16\frac{1}{2}$

If the annual interest rate remains at 10%, then Nelson can retire with \$100,000 in $16\frac{1}{2}$ years.

TRY YOUR SKILLS

Use a calculator to find the amount of interest and the new balance that will accumulate over two years on the following principal amounts at the given interest rate compounded as shown.

	Principal	Interest Rate	How Often Compounded	Interest Earned	New Balance
1.	\$1000	6%	Annually	First period: ______	______
				Second period: ______	______
2.	\$1000	6%	Semiannually	First period: ______	______
				Second period: ______	______
				Third period: ______	______
				Fourth period: ______	______

Suppose that in 1776, Benjamin Franklin and other founders of our nation each decided to invest a small amount of money for their descendants. Assume that they were able to find a bank willing to pay 4% interest, compounded semiannually for the indefinite future. Use the compound interest formula $B = p(1 + r)^n$ to find the amount that each investment will be worth after the indicated number of years have passed. Remember that for twice-a-year compounding the number of periods is twice the number of years and the interest rate for each 6-month period is $0.04 \div 2$, or 0.02.

3. Thomas Jefferson invested $100.00 for 30 years.
4. Paul Revere invested $50.00 for 90 years.
5. John Hancock invested $20.00 for 200 years.
6. Use the Rule of 72 and your calculator to find the year in which $1 that Benjamin Franklin invested in the year 1776 at 8% compounded quarterly would have become worth over $2000. Organize your results in a table like the one below.

Doublings	Years Passed	Balance
1		
2		
3		
4		
5		
6		

7. Assume that Nelson has $25,000 to invest and that he expects to earn 9% per year on his investment. The interest will be compounded quarterly. Use the Rule of 72 to approximate how long it will take for Nelson's money to double repeatedly until it has grown to more than 1 million dollars.
8. Use the compound interest formula and a graphing calculator as in Skill 4 to check your answer to Exercise 7. Which method do you think is easier? Which is more accurate? Tell which method you prefer, and give a reason for your answer.

EXERCISE YOUR SKILLS

KEY TERMS

compound interest
compounded quarterly
compounded semiannually
Rule of 72

Fill in the missing numbers. (Do not write in your book.)

1. Interest that is compounded annually is paid every __?__ months.
2. Interest that is compounded semiannually is paid every __?__ months.
3. Interest that is compounded quarterly is paid every __?__ months.
4. 5% interest compounded annually means __?__% paid every __?__ months.
5. 6% interest compounded quarterly means __?__% paid every __?__ months.

Use a calculator to find the amount of interest and the new balance that will accumulate over two years on the following principal amount at the given interest rate compounded as shown. The first quarter is done for you as an example.

	Principal	Interest Rate	How Often Compounded	Interest Earned	New Balance
6.	$1000	6%	Quarterly	First period: $15	____
7.				Second period: ____	____
8.				Third period: ____	____
9.				Fourth period: ____	____
10.				Fifth period: ____	____
11.				Sixth period: ____	____
12.				Seventh period: ____	____
13.				Eighth period: ____	____

Suppose that in 1789, John Adams invested $7.50 in a bank at 4.5% compounded semiannually and stipulated in his will that the money was to be allowed to accumulate in the account after his death. Use the compound interest formula to find the value of the account in each case.

14. After 50 years **15.** After 70 years **16.** After 100 years

17. In Exercise 16, suppose that the interest rate is 6% instead of 4.5%. Find the value of the account after 100 years. Compare the two results. What conclusion do you draw from them?

Nelson has invested $5000 in a CD. The interest is compounded annually. Use the Rule of 72 to find how many years it will take for the investment to grow to $40,000 at the given rates of interest.

18. 2% **19.** 6% **20.** 8% **21.** 10%

In the year that Nelson was born, Nelson's father invested $15,000 in an investment fund that has been paying 8% interest compounded quarterly. Use the graphing method outlined in Skill 4 to determine how long it will take for the $15,000 to grow enough to pay for Nelson's costs of attending college. Nelson estimates the costs of several colleges to be as given below.

22. $75,000 **23.** $45,000 **24.** $37,500 **25.** $52,500

26. Nelson plans to begin college at age 18. Which of the above amounts will be impossible to acquire by then?

27. At the age of 19, Robert placed $2000 into an investment at 6% compounded annually and added no money to the investment for the rest of his life. His friend Amos, who was the same age, waited until he was 30 to begin his investment program by also investing $2000 at 6% compounded annually. Later, at age 40, Amos invested another $2000 at 6% compounded annually, and still later, at age 50, he invested the final $2000 of his $6000 investment on the same interest terms. Use the compound interest formula to show that at age 60, when they both retire next year, Robert's investment will have grown to a larger amount than Amos's. How much will each investment be worth?

MIXED REVIEW

Find the commission earned and the total monthly earnings for each salesperson.

1. Paul, with a monthly base salary of $900 and a commission rate of 9% on sales of $6975

2. Gabe, with a monthly base salary of $2500 and a commission rate of 7% on sales of $7475

Suppose your bank statement shows a closing balance of $236.50 and the following are not on the statement: deposit: $80.00; check: $18.37; ATM withdrawal: $110.00. There is a service charge of $1.50 on the statement but not in your checkbook.

3. What amounts must be added to the bank statement balance?

4. What amounts must be subtracted from the bank statement balance?

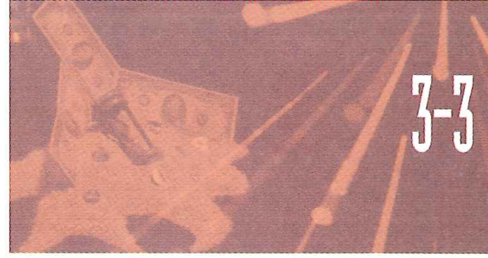
5. What should be the balance in your checkbook before you do a reconciliation?

6. What should be the checkbook balance after you do a reconciliation?

7. Pauline thinks she will write about 120 checks a month in her new business. She expects to maintain an average balance of $1500. Which of the following banks offers her the better deal? Explain your answer.

Bank A: interest of 0.035% paid on the average balance and a charge of $0.06 for each check.

Bank B: no interest paid on the checking account and no charge for checks but a monthly service charge of $6.



3-3 THE FEDERAL RESERVE: THE BANK'S BANK

Every year for as long as she can remember, Olivia has been receiving a birthday gift from her Aunt Millicent. Every November, a $20 check travels from California to Olivia's Indiana home. Last year Olivia's mother flew to California and stayed three weeks when Aunt Millicent had surgery.

Olivia knew that while in California, Mom needed more money than she had taken with her and that she was able to cash a check on an Indiana bank at Aunt Millicent's bank. Olivia thought that it was amazing that the California bank could somehow take the cash that Mom had in the bank in Indiana and give it to Mom in California in a matter of minutes. Dad said that it had something to do with the "Federal Reserve" and "computers" and declared that "computers are taking over the world." Dad has a tendency to complain. Many times, he has told Olivia how much more you could buy with $20 when he was a kid.

Olivia herself has noticed that Aunt Millicent's birthday check doesn't buy as much as it used to. Olivia took that as a sign that she was growing up, since the larger-size clothes that she now needs to buy cost more than the clothes she wore when she was little. The way her father talks about inflation, taxes, government spending, and cost-of-living increases reminds her of something she heard about the Federal Reserve System in economics class. Olivia decided to find out more about how the Federal Reserve System works.

OBJECTIVES: *In this lesson, we will help Olivia to:*

- *Describe the organization of the Federal Reserve System.*
- *List four important functions of the Federal Reserve System.*
- *Describe how the Federal Reserve System controls monetary policy.*
- *Observe and calculate the multiplier effect on the nation's money supply.*

COMPOUND INTEREST

Just as Maria discovered that interest rates vary in different institutions, Nelson found that the way in which interest is calculated can vary, too.

The interest rate is always expressed as an annual percent. This means that if Maria kept \$1000 in a CD for one year with an interest rate of 5%, she would earn \$50 in simple interest. If p represents the principal, r represents the annual rate of interest, and t represents the time in years, then the interest i is found from the simple interest formula.

$$i = prt$$
$$i = 1000 \cdot 0.05 \cdot 1$$
$$i = 50$$

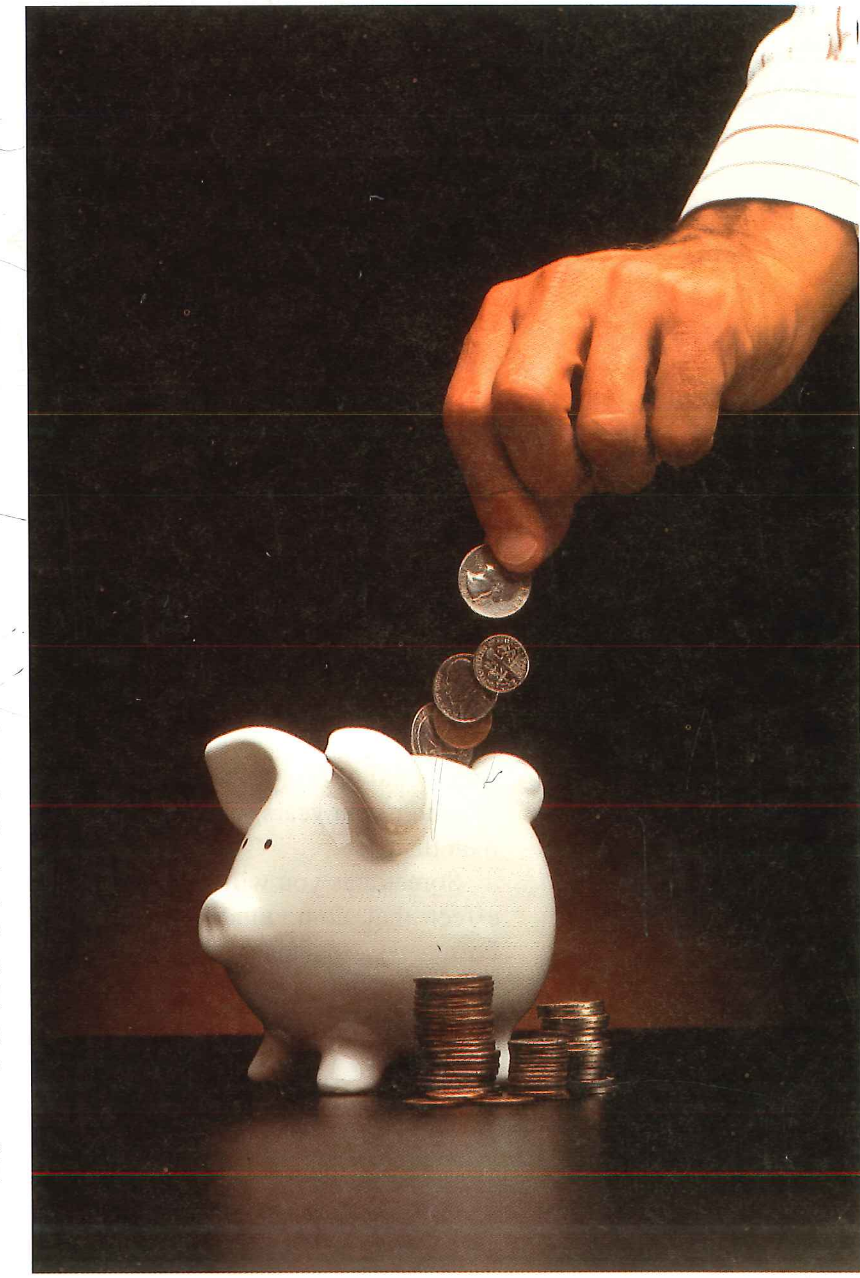
Her simple interest is \$50.

If Maria were to earn simple interest on her investment of \$1000 for a second year, then she would earn $1000 \cdot 0.05 \cdot 1$, or \$50 in the second year also. However, in practice, interest is not calculated that way. All savings institutions pay **compound interest,** that is, interest on the principal and on *the previously paid interest,* assuming that the interest is left in the account. In the second year, Maria will receive

$1050 \cdot 0.05 \cdot 1$, or \$52.50

in interest if the compounding is done only once a year.

However, most banks compound at least *semiannually.* This means that at least twice a year the interest earned is added to the previous balance, so the principal balance on which interest is paid grows a little faster than if the compounding were done only once a year. Some banks pay compound interest on whole dollar amounts only. But most large banks now compute interest on *exact amounts, compounded daily.* In this book, compound interest will be figured on exact amounts.



If Maria received 5% **compounded semiannually** on her original \$1000, then her interest for the first year would be calculated as follows. To find half of the yearly interest, divide the yearly interest rate by 2. If the yearly rate is 5%, divide by 2 to obtain a semiannual rate of $2\frac{1}{2}\%$.

$1000 \cdot 0.025 = 25.00$	First 6 months: interest on initial principal
$1025 \cdot 0.025 = \underline{25.63}$	Second 6 months: interest on principal and interest
50.63	Total interest for 1 year

Of course, the interest has been rounded to the nearest cent. Maria earned 63 cents more than she would have if the interest had not been compounded.

You can calculate the interest that is **compounded quarterly** in a similar manner. Since the interest is calculated and paid four times a year (every three months), the quarterly interest rate is found by dividing the yearly rate by 4. If the yearly rate is 5%, divide by 4 to obtain a quarterly rate of 1.25%.

Increasing the frequency of compounding does not greatly increase the amount of interest that you actually get. The table below shows the total interest that you would receive over ten years on \$1000 at a rate of 5% under several compounding methods.

	Annually	Semiannually	Quarterly	Continuously
Interest received	\$628.89	\$638.61	\$643.62	\$648.72

Even if your original \$1000 were compounded *continuously* (every split second), in ten years you would gain only about \$20 more than by compounding annually. So it does not pay to put a lot of emphasis on the frequency of compounding when you choose a bank. However, *all* of the compounding methods give you much more money than you would receive if the interest were calculated just as simple interest, as shown below:

$i = prt$	
$i = 1000 \cdot 0.05 \cdot 10$	5% interest paid on only the original \$1000
$i = 500$	10 years of simple interest

You would get only \$500 in simple interest at 5% on your original \$1000 at the end of ten years. Compound interest would pay you at least \$128 more than that.

Sometimes you will want to have an approximate idea of the long-range effect that compounding will have on your savings or investments. Fortunately, there is an easy calculation that you can perform to help you do this. It is explained at the top of the next page.