

§ 11-6 NATURAL LOGARITHMS

GOALS: • FIND THE NATURAL LOG OF #'S

- SOLVE EQUATION'S AND INEQUALITIES
USING NATURAL LOGS
- SOLVE REAL WORLD PROBLEMS w/
NATURAL LOGS

NATURAL LOGARITHM — LOG WITH BASE e

$$\log_e x = \ln x$$

[LN] IS ALSO A

KEY ON MOST
CALCULATORS

* RECALL $e \approx 2.71828$, IT IS
A POSITIVE # AND
ALL OF THE PROPERTIES OF
LOGS HOLD TRUE

$$\ln e = x$$

$\log_e e = x$ WRITTEN AS AN EXPONENTIAL

$$e^x = e$$

$$e^x = e^1 \therefore x = 1$$

so $\boxed{\ln e = 1}$

ANTILOGARITHM OF A NATURAL LOGARTHM

IF $\ln x = a$, THEN $x = \text{antiln } a$

* INVERSE OF A LOG IS AN EXPONENTIAL.

$x = \text{antiln } a$ IS THE SAME AS

$$x = e^a$$

ON THE CALCULATOR USE

[2ND] [LN] ...

e^x

EXAMPLE

CONVERT $\log_5 295$ TO A NATURAL LOG
AND EVALUATE.

$$\log_5 295 = \frac{\log_e 295}{\log_e 5}$$

* CHANGE OF
BASE FORMULA

$$= \frac{\ln 295}{\ln 5}$$

* $\log_e = \ln$

* USE CALCULATOR

TO FIND
 $\ln 295$?
 $\ln 5$

$$= \boxed{3.5335}$$

* THIS IS NOT EASIER THAN LOG JUST
ANOTHER WAY.

SOLVE $7.2 = -28.8 \ln x$

$$\frac{7.2}{-28.8} = \frac{-28.8 \ln x}{-28.8}$$

$$-\frac{1}{4} = \ln x$$

$$-\frac{1}{4} = \ln x$$

$$\text{anti} \ln \left(-\frac{1}{4} \right) = x \quad \text{or} \quad e^{-\frac{1}{4}} = x$$
$$\boxed{x = 1.7788}$$

SOLVE $5^{2x} = 7^{x+1}$ USING NATURAL LOGS

$$5^{2x} = 7^{x+1}$$
$$\ln 5^{2x} = \ln 7^{x+1}$$

$$(2x)\ln 5 = (x+1)\ln 7$$

$$(2x)1.6094 = (x+1)1.9459$$

$$3.2189x = 1.9459x + 1.9459$$

$$1.2729x = 1.9459$$

$$\boxed{x = 1.5286}$$

SOLVE $4^{x-1} < 12$

$$\ln 4^{x-1} < \ln 12$$

$$(x-1)\ln 4 < \ln 12$$
$$(x-1) < \frac{\ln 12}{\ln 4}$$

$$x < 1 + \frac{\ln 12}{\ln 4}$$

* TAKE LN OF
BOTH SIDES

* POWER PROPERTY

* USE CALCULATOR
TO FIND $\ln 5$?
 $\ln 7$

* SOLVE WITH ALGEBRA

$$x < 1 + \frac{\ln 12}{\ln 4}$$

$$x < 1 + \frac{2.4849}{1.3862}$$

$$x < 1 + 1.79248125$$

$$\boxed{x < 2.7925}$$

APPLICATIONS

CONTINUOUSLY COMPOUNDED INTEREST

$$A = P e^{rt}$$

A = Amount

P = principal investment

r = annual interest rate

t = time

How long will it take a \$5000 investment to be worth \$1,000,000 if it earns 8% annual interest compounded continuously?

$$A = P e^{rt}$$
$$\frac{1000000}{5000} = \frac{5000}{5000} e^{(.08)t}$$

$$200 = e^{.08t}$$

$$\ln 200 = \ln e^{.08t}$$

$$\ln 200 = .08t (\ln e)$$

$$\frac{5.2983}{.08} = \frac{.08t}{.08} (1)$$

$$\boxed{t = 66.2 \text{ years}}$$

THE SPREAD OF A RUMOR

$$P = \frac{P_0 e^{kt}}{1 - P_0(1 - e^{kt})}$$

P_0 = FRACTION OF PEOPLE WHO HAVE HEARD RUMOR AFTER t days
 t = # OF DAYS
 k = A CONSTANT

Example

A STUDY DETERMINED THAT AT $t=0$, 10% OF THE PEOPLE HEARD A RUMOR AND AFTER 7 DAYS 75% HEARD IT.

How long until 95% will hear the rumor?

$$.75 = \frac{.10e^{7k}}{1 - .10(1 - e^{7k})}$$

$$.75(1 - .10(1 - e^{7k})) = .10e^{7k}$$

$$.75(1 - .10 + .10e^{7k}) = .10e^{7k}$$

$$.75(.9 + .10e^{7k}) = .10e^{7k}$$

$$.675 + .075e^{7k} = .10e^{7k}$$

$$-.075e^{7k} \quad -.075e^{7k}$$

$$\frac{0.675}{0.025} = \frac{0.025e^{7k}}{0.025}$$

$$27 = e^{7k}$$

$$\ln 27 = \ln e^{7k}$$

$$\ln 27 = 7k(\ln e)$$

$$3.2958 = 7k(1)$$

$$k = 0.470833838$$

USING THE CONSTANT K , now we can find out when 95% of the population will know the rumor.

$$0.95 = \frac{0.10e^{0.4708t}}{1 - 0.10(1 - e^{-0.4708t})}$$

$$0.95(1 - 0.10(1 - e^{-0.4708t})) = 0.10e^{0.4708t}$$

$$0.95(1 - 0.10 + 0.10e^{0.4708t}) = 0.10e^{0.4708t}$$

$$0.95(0.9 + 0.10e^{0.4708t}) = 0.10e^{0.4708t}$$

$$0.855 + 0.095e^{0.4708t} = 0.10e^{0.4708t}$$

$$- 0.095e^{0.4708t} - 0.095e^{0.4708t}$$

$$\underline{0.855} = \frac{.005e^{0.4708t}}{.005}$$

$$171 = e^{0.4708t}$$

$$\ln 171 = \ln e^{0.4708t}$$

$$5.14166 = .04708t (\ln e)$$

$$5.14166 = .04708t (1)$$

$$10.920 = t$$

or 11 days

P. 736-737



18-34 EVEN, 36-47, 54-60

