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## Study Guide

## Natural Logarithms

Logarithms with base $e$ are called natural logarithms and are usually written $\ln \boldsymbol{x}$. Logarithms with a base other than $e$ can be converted to natural logarithms using the change of base formula. The antilogarithm of a natural logarithm is written antiln $\boldsymbol{x}$. You can use the properties of logarithms and antilogarithms to simplify and solve exponential and logarithmic equations or inequalities with natural logarithms.

## Example 1 Convert $\log _{4} 381$ to a natural logarithm and

 evaluate.$$
\begin{aligned}
\log _{a} n & =\frac{\log _{b} n}{\log _{b} a} & & \\
\log _{4} 381 & =\frac{\log _{e} 381}{\log _{e} 4} & & a=4, b=e, n=381 \\
& =\frac{\ln 381}{\ln 4} & & \log _{e} x=\ln x \\
& \approx 4.2868 & & \text { Use a calculator. }
\end{aligned}
$$

So, $\log _{4} 381$ is about 4.2868 .
Example 2 Solve $3.75=-7.5 \ln \boldsymbol{x}$.

$$
\begin{aligned}
3.75 & =-7.5 \ln x & & \\
-0.5 & =\ln x & & \text { Divide each side by }-7.5 \\
\text { antiln }(-0.5) & =x & & \text { Take the antilogarithm of each side. } \\
0.6065 & \approx x & & \text { Use a calculator. }
\end{aligned}
$$

The solution is about 0.6065 .

Example 3 Solve each equation or inequality by using natural logarithms.
a. $4^{3 x}=6^{x+1}$

$$
4^{3 x}=6^{x+1}
$$

$\ln 4^{3 x}=\ln 6^{x+1}$
$3 x \ln 4=(x+1) \ln 6$ $3 x(1.3863)=(x+1)(1.7918)$

Take the natural logarithm of each side.
$\ln a^{n}=n \ln a$
$4.1589 x=1.7918 x+1.7918$
$2.3671 x=1.7918$

$$
x \approx 0.7570
$$

b. $25>e^{0.2 t}$
$25>e^{0.2 t}$
$\ln 25>\ln e^{0.2 t} \quad$ Take the natural logarithm of each side.
$\ln 25>0.2 t \ln e \ln a^{n}=n \ln a$
$3.2189>0.2 t \quad$ Use a calculator.
$16.0945>t$
Thus, $t<16.0945$
$\qquad$
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$\qquad$

## Evaluate each expression.

6. antiln -1.62

Convert each logarithm to a natural logarithm and evaluate.
7. $\log _{7} 94$
8. $\log _{5} 256$
9. $\log _{9} 0.712$

Use natural logarithms to solve each equation or inequality.
10. $6^{x}=42$
11. $7^{x}=4^{x+3}$
12. $1249=175 e^{-0.04 t}$
13. $10^{x+1}>3^{x}$
14. $12<e^{0.048 y}$
15. $8.4<e^{t-2}$
16. Banking Ms. Cubbatz invested a sum of money in a certificate of deposit that earns $8 \%$ interest compounded continuously. The formula for calculating interest that is compounded continuously is $A=P e^{r t}$. If Ms. Cubbatz made the investment on January 1, 1995, and the account was worth $\$ 12,000$ on January 1, 1999, what was the original amount in the account?

