

Study Guide

Common Logarithms

Logarithms with base 10 are called **common logarithms**.

The change of base formula, $\log_a n = \frac{\log_b n}{\log_b a}$, where a , b , and n

are positive numbers and neither a nor b is 1, allows you to evaluate logarithms in other bases with a calculator.

Logarithms can be used to solve **exponential equations**.

Example 1 Evaluate each expression.

a. $\log 8(3)^2$

$$\begin{aligned}\log 8(3)^2 &= \log 8 + 2 \log 3 & \log ab &= \log a + \log b, \log b^n = n \log b \\ &\approx 0.9031 + 2(0.4771) & \text{Use a calculator.} \\ &\approx 0.9031 + 0.9542 \\ &\approx 1.8573\end{aligned}$$

b. $\log \frac{15^3}{7}$

$$\begin{aligned}\log \frac{15^3}{7} &= 3 \log 15 - \log 7 & \log \frac{a}{b} &= \log a - \log b, \log a^m = m \log a \\ &\approx 3(1.1761) - 0.8451 & \text{Use a calculator.} \\ &\approx 3.5283 - 0.8451 \\ &\approx 2.6832\end{aligned}$$

Example 2 Find the value of $\log_8 2037$ using the change of base formula.

$$\begin{aligned}\log_8 2037 &= \frac{\log_{10} 2037}{\log_{10} 8} & \log_a n &= \frac{\log_b n}{\log_b a} \\ &\approx \frac{3.3090}{0.9031} & \text{Use a calculator.} \\ &\approx 3.6641\end{aligned}$$

Example 3 Solve $7^{2x} = 93$.

$$\begin{aligned}7^{2x} &= 93 \\ \log 7^{2x} &= \log 93 & \text{Take the logarithm of each side.} \\ 2x \log 7 &= \log 93 & \log_b m^p &= p \cdot \log_b m \\ 2x &= \frac{\log 93}{\log 7} & \text{Divide each side by } \log 7. \\ 2x &\approx 2.3293 & \text{Use a calculator.} \\ x &\approx 1.1646\end{aligned}$$

Practice

Common Logarithms

Given that $\log 3 = 0.4771$, $\log 5 = 0.6990$, and $\log 9 = 0.9542$, evaluate each logarithm.

1. $\log 300,000$
2. $\log 0.0005$
3. $\log 9000$
4. $\log 27$
5. $\log 75$
6. $\log 81$

Evaluate each expression.

7. $\log 66.3$
8. $\log \frac{17^4}{5}$
9. $\log 7(4^3)$

Find the value of each logarithm using the change of base formula.

10. $\log_6 832$
11. $\log_{11} 47$
12. $\log_3 9$

Solve each equation or inequality.

13. $8^x = 10$
14. $2.4^x \leq 20$
15. $1.8^{x-5} = 19.8$
16. $3^{5x} = 85$
17. $4^{2x} > 25$
18. $3^{2x-2} = 2^x$

19. **Seismology** The intensity of a shock wave from an earthquake is given by the formula $R = \log_{10} \frac{I}{I_0}$, where R is the magnitude, I is a measure of wave energy, and $I_0 = 1$. Find the intensity per unit of area for the following earthquakes.

- a. Northridge, California, in 1994, $R = 6.7$
- b. Hector Mine, California, in 1999, $R = 7.1$