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## Study Guide

## The Number e

The number $\boldsymbol{e}$ is a special irrational number with an approximate value of 2.718 to three decimal places. The formula for exponential growth or decay is $N=N_{0} e^{k t}$, where $N$ is the final amount, $N_{0}$ is the initial amount, $k$ is a constant, and $t$ is time. The equation $A=P e^{r t}$, where $P$ is the initial amount, $A$ is the final amount, $r$ is the annual interest rate, and $t$ is time in years, is used for calculating interest that is compounded continuously.

## Example 1 Demographics The population of Dubuque, Iowa, declined at a rate of $0.4 \%$ between 1997 1998. In 1998, the population was 87,806 . <br> a. Let $t$ be the number of years since 1998 and write a function to model the population.

b. Suppose that the rate of decline remains steady at $0.4 \%$. Find the projected population of Dubuque in 2010.
a. $y=n e^{k t}$
$y=87,806 e^{-0.004 t} \quad n=87,806 ; k=-0.004$
b. In 2010, it will have been 2010 - 1998 or 12 years since the initial population figure. Thus, $t=12$.

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\begin{array}{ll}
y=87,806 e^{-0.004 t} & \\
y=87,806 e^{-0.004(12)} & t=12 \\
y \approx 83690.86531 & \text { Use a calculator } .
\end{array}
$$

Given a population of 87,806 in 1998 and a steady rate of decline of $0.4 \%$, the population of Dubuque, Iowa, will be approximately 83,691 in 2010.

Example 2 Finance Compare the balance after 10 years of a $\$ 5000$ investment earning $8.5 \%$ interest compounded continuously to the same investment compounded quarterly.
In both cases, $P=5000, r=0.085$, and $t=10$. When the interest is compounded quarterly, $n=4$. Use a calculator to evaluate each expression.

## Continuously

$A=P e^{r t}$
$A=5000 e^{(0.085)(10)}$
$A=11,698.23$

## Quarterly

$$
\begin{aligned}
& A=P\left(1+\frac{r}{n}\right)^{n t} \\
& A=5000\left(1+\frac{0.085}{4}\right)^{4 \cdot 10} \\
& A=11,594.52
\end{aligned}
$$

You would earn $\$ 11,698.23-\$ 11,594.52=\$ 103.71$ more by choosing the account that compounds continuously.
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## Practice

## The Number e

1. Demographics In 1995, the population of Kalamazoo, Michigan, was 79,089. This figure represented a $0.4 \%$ annual decline from 1990.
a. Let $t$ be the number of years since 1995 and write a function that models the population in Kalamazoo in 1995.
b. Predict the population in 2010 and 2015. Assume a steady rate of decline.
2. Biology Suppose a certain type of bacteria reproduces according to the model $P(t)=100 e^{0.271 t}$, where $t$ is time in hours.
a. At what rate does this type of bacteria reproduce?
b. What was the initial number of bacteria?
c. Find the number of bacteria at $P(5), P(10), P(24)$, and $P(72)$. Round to the nearest whole number.
3. Finance Suppose Karyn deposits $\$ 1500$ in a savings account that earns $6.75 \%$ interest compounded continuously. She plans to withdraw the money in 6 years to make a $\$ 2500$ down payment on a car. Will there be enough funds in Karyn's account in 6 years to meet her goal?
4. Banking Given the original principal, the annual interest rate, the amount of time for each investment, and the type of compounded interest, find the amount at the end of the investment.
a. $P=\$ 1250, r=8.5 \%, t=3$ years, semiannually
b. $P=\$ 2575, r=6.25 \%, t=5$ years 3 months, continuously
