Study Guide

Exponential Functions

Functions of the form $y = b^x$, in which the base *b* is a positive real number and the exponent is a variable, are known as **exponential functions.** Many real-world situations can be modeled by exponential functions. The equation $N = N_0(1 + r)^t$, where *N* is the final amount, N_0 is the initial amount, *r* is the rate of growth or decay, and *t* is time, is used for modeling exponential growth. The compound interest equation is $A = P(1 + \frac{r}{n})^{nt}$, where *P* is the principal or initial investment, *A* is the final amount of the investment, *r* is the annual interest rate, *n* is the number of times interest is compounded each year, and *t* is the number of years.

Example 1 Graph $y < 2^{-x}$.

First, graph $y = 2^{-x}$. This graph is a function, since there is a unique *y*-value for each *x*-value.

x	-3	-2	-1	0	1	2	3	4
2 ^{-x}	8	4	2	1	<u>1</u> 2	$\frac{1}{4}$	<u>1</u> 8	<u>1</u> 16

Since the points on this curve are not in the solution of the inequality, the graph of $y = 2^{-x}$ is shown as a dashed curve.



Then, use (0, 0) as a test point to determine which area to shade. $y < 2^{-x}$

- $0 < 2^{0}$
- 0 < 1

Since (0,0) satisfies the inequality, the region that contains (0,0) should be shaded.

Example 2 Biology Suppose a researcher estimates that the initial population of a colony of cells is 100. If the cells reproduce at a rate of 25% per week, what is the expected population of the colony in six weeks?

$$\begin{split} N &= N_0 (1+r)^t \\ N &= 100(1+0.25)^6 \quad N_0 = 100, \, r = 0.25, \, t = 6 \\ N &\approx 381.4697266 \qquad Use \; a \; calculator. \\ \text{There will be about } 381 \; \text{cells in the colony in 6 weeks.} \end{split}$$

Example 3 Finance Determine the amount of money in a money market account that provides an annual rate of 6.3% compounded quarterly if \$1700 is invested and left in the account for eight years.

$$\begin{split} A &= P \Big(1 + \frac{r}{n} \Big)^{nt} \\ A &= 1700 \Big(1 + \frac{0.063}{4} \Big)^{4 \cdot 8} \qquad P = 1700, \, r = 0.063, \, n = 4, \, t = 8 \\ A &\approx 2803.028499 \qquad Use \, a \, calculator. \\ \text{After 8 years, the $1700 investment will have a value of $2803.03.} \end{split}$$





Practice

Exponential Functions

Graph each exponential function or inequality.



- 5. Demographics An area in North Carolina known as The Triangle is principally composed of the cities of Durham, Raleigh, and Chapel Hill. The Triangle had a population of 700,000 in 1990. The average yearly rate of growth is 5.9%. Find the projected population for 2010.
- **6.** *Finance* Determine the amount of money in a savings account that provides an annual rate of 4% compounded monthly if the initial investment is \$1000 and the money is left in the account for 5 years.
- 7. *Investments* How much money must be invested by Mr. Kaufman if he wants to have \$20,000 in his account after 15 years? He can earn 5% compounded quarterly.