

Study Guide

Hyperbolas

The standard form of the equation of a **hyperbola** is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \text{ when the transverse axis is horizontal, and}$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 \text{ when the transverse axis is vertical. In both cases, } b^2 = c^2 - a^2.$$

Example Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $25x^2 - 9y^2 + 100x - 54y - 206 = 0$. Then graph the equation.

Write the equation in standard form.

$$25x^2 - 9y^2 + 100x - 54y - 206 = 0$$

$$25(x^2 + 4x + ?) - 9(y^2 + 6y + ?) = 206 + ? + ? \quad \begin{array}{l} \text{GCF of } x \text{ terms is } 25; \\ \text{GCF of } y \text{ terms is } 9. \end{array}$$

$$25(x^2 + 4x + 4) - 9(y^2 + 6y + 9) = 206 + 25(4) + (-9)(9) \quad \text{Complete the square.}$$

$$25(x + 2)^2 - 9(y + 3)^2 = 225 \quad \text{Factor.}$$

$$\frac{(x+2)^2}{9} - \frac{(y+3)^2}{25} = 1 \quad \text{Divide each side by 225.}$$

From the equation, $h = -2$, $k = -3$, $a = 3$, $b = 5$, and $c = \sqrt{34}$. The center is at $(-2, -3)$.

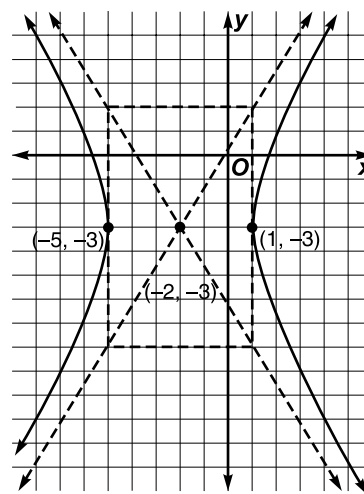
Since the x terms are in the first expression, the hyperbola has a horizontal transverse axis.

The vertices are at $(h \pm a, k)$ or $(1, -3)$ and $(-5, -3)$.

The foci are at $(h \pm c, k)$ or $(-2 \pm \sqrt{34}, -3)$.

The equations of the asymptotes are $y - k = \pm \frac{b}{a}(x - h)$ or $y + 3 = \pm \frac{5}{3}(x + 2)$.

Graph the center, the vertices, and the rectangle guide, which is $2a$ units by $2b$ units. Next graph the asymptotes. Then sketch the hyperbola.

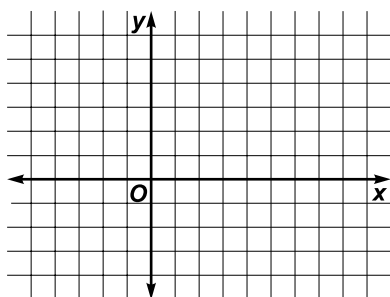


Practice

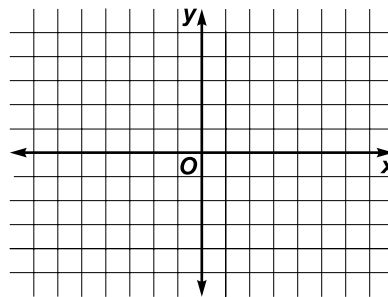
Hyperbolas

For each equation, find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of its graph. Then graph the equation.

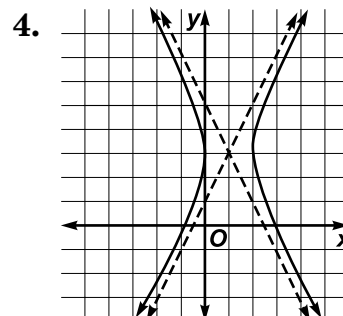
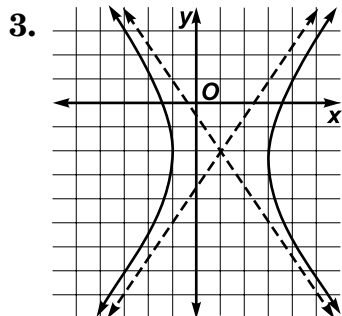
1. $x^2 - 4y^2 - 4x + 24y - 36 = 0$



2. $y^2 - 4x^2 + 8x = 20$



Write the equation of each hyperbola.



5. Write an equation of the hyperbola for which the length of the transverse axis is 8 units, and the foci are at $(6, 0)$ and $(-4, 0)$.

6. **Environmental Noise** Two neighbors who live one mile apart hear an explosion while they are talking on the telephone. One neighbor hears the explosion two seconds before the other. If sound travels at 1100 feet per second, determine the equation of the hyperbola on which the explosion was located.