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Study Guide

Ellipses

The standard form of the equation of an **ellipse** is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ when the **major axis** is horizontal. In this case, a^2 is in the denominator of the x term. The standard form is $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ when the major axis is vertical. In this case, a^2 is in the denominator of the y term. In both cases, $c^2 = a^2 - b^2$.

Example **Find the coordinates of the center, the foci, and the vertices of the ellipse with the equation $4x^2 + 9y^2 + 24x - 36y + 36 = 0$. Then graph the equation.**

First write the equation in standard form.

$$4x^2 + 9y^2 + 24x - 36y + 36 = 0$$

$$4(x^2 + 6x + ?) + 9(y^2 - 4y + ?) = -36 + ? + ? \quad \textit{GCF of x terms is 4; GCF of y terms is 9.}$$

$$4(x^2 + 6x + 9) + 9(y^2 - 4y + 4) = -36 + 4(9) + 9(4) \quad \textit{Complete the square.}$$

$$4(x+3)^2 + 9(y-2)^2 = 36 \quad \textit{Factor.}$$

$$\frac{(x+3)^2}{9} + \frac{(y-2)^2}{4} = 1 \quad \textit{Divide each side by 36.}$$

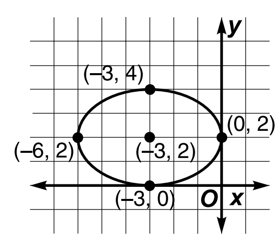
Now determine the values of a, b, c, h, and k. In all ellipses, $a^2 > b^2$. Therefore, $a^2 = 9$ and $b^2 = 4$. Since a^2 is the denominator of the x term, the major axis is parallel to the x-axis.

$a = 3 \quad b = 2 \quad c = \sqrt{a^2 - b^2}$ or $\sqrt{5} \quad h = -3 \quad k = 2$

center: $(-3, 2)$ — (h, k)
foci: $(-3 \pm \sqrt{5}, 2)$ — $(h \pm c, k)$

major axis vertices:
$(0, 2)$ and $(-6, 2)$ — $(h \pm a, k)$

minor axis vertices:
$(-3, 4)$ and $(-3, 0)$ — $(h, k \pm b)$



Graph these ordered pairs. Then complete the ellipse.

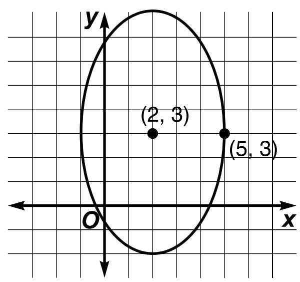
10-3

Practice

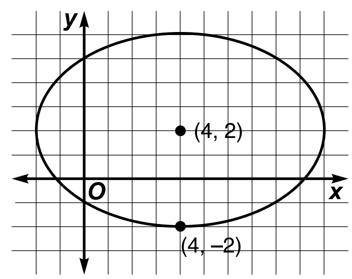
Ellipses

Write the equation of each ellipse in standard form. Then find the coordinates of its foci.

1.

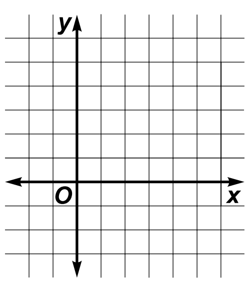


2.

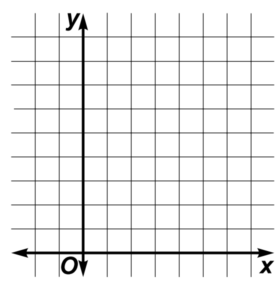


For the equation of each ellipse, find the coordinates of the center, foci, and vertices. Then graph the equation.

3. $4x^2 + 9y^2 - 8x - 36y + 4 = 0$



4. $25x^2 + 9y^2 - 50x - 90y + 25 = 0$



Write the equation of the ellipse that meets each set of conditions.

5. The center is at (1, 3), the major axis is parallel to the y-axis, and one vertex is at (1, 8), and $b = 3$.

6. The foci are at $(-2, 1)$ and $(-2, -7)$, and $a = 5$.

7. ***Construction*** A semi elliptical arch is used to design a headboard for a bed frame. The headboard will have a height of 2 feet at the center and a width of 5 feet at the base. Where should the craftsman place the foci in order to sketch the arch?

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Enrichment

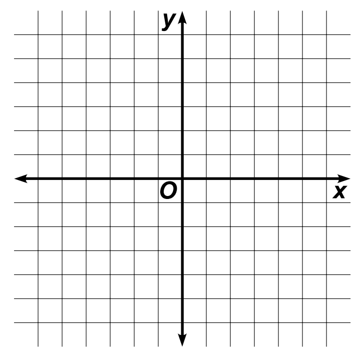
Superellipses

The circle and the ellipse are members of an interesting family of curves that were first studied by the French physicist and mathematician Gabriel Lamé (1795-1870). The general equation for the family is

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1, \text{ with } a \neq 0,\ b \neq 0, \text{ and } n > 0.$$

For even values of n greater than 2, the curves are called **superellipses**.

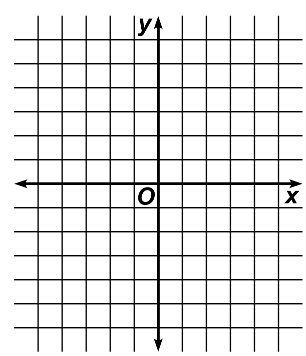
1. Consider two curves that are *not* superellipses. Graph each equation on the grid at the right. State the type of curve produced each time.



a. $\left|\frac{x}{2}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$

b. $\left|\frac{x}{3}\right|^2 + \left|\frac{y}{2}\right|^2 = 1$

2. In each of the following cases you are given values of a, b, and n to use in the general equation. Write the resulting equation. Then graph. Sketch each graph on the grid at the right.



a. $a = 2, b = 3, n = 4$

b. $a = 2, b = 3, n = 6$

c. $a = 2, b = 3, n = 8$

3. What shape will the graph of $\left|\frac{x}{2}\right|^n + \left|\frac{y}{3}\right|^n = 1$ approximate for greater and greater even, whole-number values of n?