NAME ______________________ DATE ______________ PERIOD ________

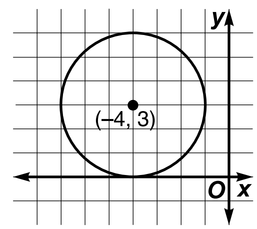
Study Guide

Circles

The standard form of the equation of a **circle** with **radius** r and **center** at (h, k) is $(x - h)^2 + (y - k)^2 = r^2$.

Example 1 **Write the standard form of the equation of the circle that is tangent to the *x*-axis and has its center at $(-4, 3)$. Then graph the equation.**

Since the circle is tangent to the x-axis, the distance from the center to the x-axis is the radius. The center is 3 units above the x-axis. Therefore, the radius is 3.



$$(x - h)^2 + (y - k)^2 = r^2 \quad \textit{Standard form}$$
$$[x - (-4)]^2 + (y - 3)^2 = 3^2 \quad (h, k) = (-4, 3) \textit{ and } r = 3$$
$$(x + 4)^2 + (y - 3)^2 = 9$$

Example 2 **Write the standard form of the equation of the circle that passes through the points at $(1, -1)$, $(5, 3)$, and $(-3, 3)$. Then identify the center and radius of the circle.**

Substitute each ordered pair (x, y) in the general form $x^2 + y^2 + Dx + Ey + F = 0$ to create a system of equations.

$$(1)^2 + (-1)^2 + D(1) + E(-1) + F = 0 \quad (x, y) = (1, -1)$$
$$(5)^2 + (3)^2 + D(5) + E(3) + F = 0 \quad (x, y) = (5, 3)$$
$$(-3)^2 + (3)^2 + D(-3) + E(3) + F = 0 \quad (x, y) = (-3, 3)$$

Simplify the system of equations.

$$D - E + F + 2 = 0$$
$$5D + 3E + F + 34 = 0$$
$$-3D + 3E + F + 18 = 0$$

The solution to the system is $D = -2$, $E = -6$, and $F = -6$.

The general form of the equation of the circle is $x^2 + y^2 - 2x - 6y - 6 = 0$.

$$x^2 + y^2 - 2x - 6y - 6 = 0$$
$$(x^2 - 2x + ?) + (y^2 - 6y + ?) = 6 \quad \textit{Group to form perfect square trinomials.}$$
$$(x^2 - 2x + 1) + (y^2 - 6y + 9) = 6 + 1 + 9 \quad \textit{Complete the square.}$$
$$(x - 1)^2 + (y - 3)^2 = 16 \quad \textit{Factor the trinomials.}$$

After completing the square, the standard form of the circle is $(x - 1)^2 + (y - 3)^2 = 16$. Its center is at $(1, 3)$, and its radius is 4.

NAME ______________________ DATE ______________ PERIOD ________

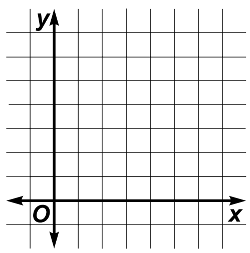
10-2

Practice

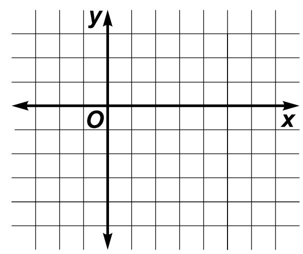
Circles

Write the standard form of the equation of each circle described. Then graph the equation.

1. center at $(3, 3)$ tangent to the x-axis

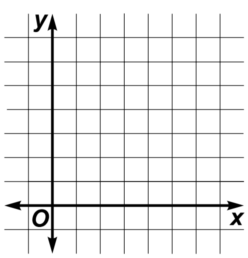


2. center at $(2, -1)$, radius 4

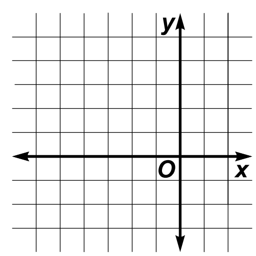


Write the standard form of each equation. Then graph the equation.

3. $x^2 + y^2 - 8x - 6y + 21 = 0$



4. $4x^2 + 4y^2 + 16x - 8y - 5 = 0$



Write the standard form of the equation of the circle that passes through the points with the given coordinates. Then identify the center and radius.

5. $(-3, -2), (-2, -3), (-4, -3)$

6. $(0, -1), (2, -3), (4, -1)$

7. ***Geometry*** A square inscribed in a circle and centered at the origin has points at $(2, 2), (-2, 2), (2, -2)$ and $(-2, -2)$. What is the equation of the circle that circumscribes the square?

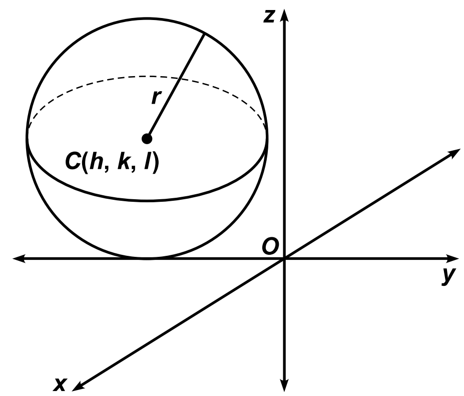
NAME ______________________ DATE ____________ PERIOD ________

Enrichment

Spheres

The set of all points in three-dimensional space that are a fixed distance r (the **radius**), from a fixed point C (the **center**), is called a **sphere**. The equation below is an algebraic representation of the sphere shown at the right.

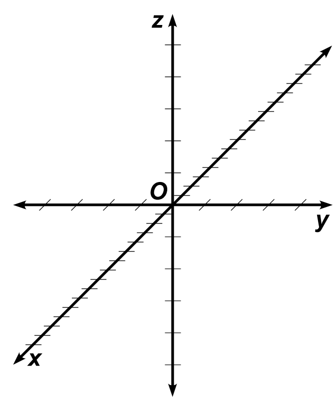
$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$



A line segment containing the center of a sphere and having its endpoints on the sphere is called a **diameter** of the sphere. The endpoints of a diameter are called **poles** of the sphere. A **great circle** of a sphere is the intersection of the sphere and a plane containing the center of the sphere.

1. If $x^2 + y^2 - 4y + z^2 + 2z - 4 = 0$ is an equation of a sphere and $(1, 4, -3)$ is one pole of the sphere, find the coordinates of the opposite pole.

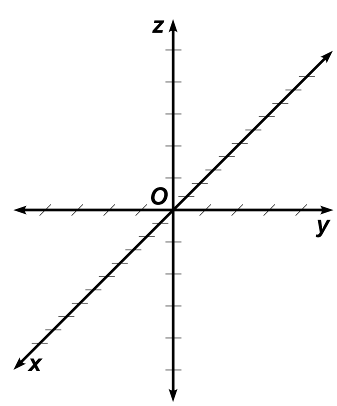
2. a. On the coordinate system at the right, sketch the sphere described by the equation $x^2 + y^2 + z^2 = 9$.



b. Is $P(2, -2, -2)$ inside, outside, or on the sphere?

c. Describe a way to tell if a point with coordinates $P(a, b, c)$ is inside, outside, or on the sphere with equation $x^2 + y^2 + z^2 = r^2$.

3. If $x^2 + y^2 + z^2 - 4x + 6y - 2z - 2 = 0$ is an equation of a sphere, find the circumference of a great circle, and the surface area and volume of the sphere.



4. The equation $x^2 + y^2 = 4$ represents a set of points in three-dimensional space. Describe that set of points in your own words. Illustrate with a sketch on the coordinate system at the right.