NAME ______________________ DATE ______________ PERIOD ________

6-1

Study Guide

Angles and Radian Measure

An angle of one complete revolution can be represented either by 360° or by 2π radians. Thus, the following formulas can be used to relate degree and **radian** measures.

Degree/Radian Conversion Formulas	1 radian = $\frac{180}{\pi}$ degrees or about 57.3° 1 degree = $\frac{\pi}{180}$ radians or about 0.017 radian

Example 1 **a. Change 36° to radian measure in terms of π.**
b. Change $-\frac{17\pi}{3}$ radians to degree measure.

a. $36° = 36° \times \frac{\pi}{180°}$
$= \frac{\pi}{5}$

b. $-\frac{17\pi}{3} = -\frac{17\pi}{3} \times \frac{180°}{\pi}$
$= -1020°$

Example 2 **Evaluate $\sin \frac{3\pi}{4}$.**

The reference angle for $\frac{3\pi}{4}$ is $\frac{\pi}{4}$. Since $\frac{\pi}{4} = 45°$, the terminal side of the angle intersects the unit circle at a point with coordinates of $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. Because the terminal side of $\frac{3\pi}{4}$ lies in Quadrant II, the x-coordinate is negative and the y-coordinate is positive. Therefore, $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$.

Example 3 **Given a central angle of 147°, find the length of its intercepted arc in a circle of radius 10 centimeters. Round to the nearest tenth.**

First convert the measure of the central angle from degrees to radians.

$147° = 147° \times \frac{\pi}{180°}$ *1 degree = $\frac{\pi}{180°}$*
$= \frac{49\pi}{60}$

Then find the length of the arc.

$s = r\theta$ *Formula for the length of an arc*

$s = 10\left(\frac{49\pi}{60}\right)$ *$r = 10$, $\theta = \frac{49\pi}{60}$*

$s \approx 25.65634$

The length of the arc is about 25.7 cm.

6-1

NAME ______________________ DATE ______________ PERIOD ________

Practice

Angles and Radian Measure

Change each degree measure to radian measure in terms of π.

1. $-250°$ **2.** $6°$ **3.** $-145°$

4. $870°$ **5.** $18°$ **6.** $-820°$

Change each radian measure to degree measure. Round to the nearest tenth, if necessary.

7. 4π **8.** $\frac{13\pi}{30}$ **9.** -1

10. $\frac{3\pi}{16}$ **11.** -2.56 **12.** $-\frac{7\pi}{9}$

Evaluate each expression.

13. $\tan \frac{\pi}{4}$ **14.** $\cos \frac{3\pi}{2}$ **15.** $\sin \frac{3\pi}{2}$

16. $\tan \frac{11\pi}{6}$ **17.** $\cos \frac{3\pi}{4}$ **18.** $\sin \frac{5\pi}{3}$

Given the measurement of a central angle, find the length of its intercepted arc in a circle of radius 10 centimeters. Round to the nearest tenth.

19. $\frac{\pi}{6}$ **20.** $\frac{3\pi}{5}$ **21.** $\frac{\pi}{2}$

Find the area of each sector, given its central angle θ and the radius of the circle. Round to the nearest tenth.

22. $\theta = \frac{\pi}{6}, r = 14$ **23.** $\theta = \frac{7\pi}{4}, r = 4$

NAME ______________________ DATE ______________ PERIOD ________

6-2

Study Guide

Linear and Angular Velocity

As a circular object rotates about its center, an object at the edge moves through an angle relative to the object's starting position. That is known as the **angular displacement,** or angle of rotation. **Angular velocity** ω is given by $\omega = \frac{\theta}{t}$, where θ is the angular displacement in radians and t is time. **Linear velocity** v is given by $v = r\frac{\theta}{t}$, where $\frac{\theta}{t}$ represents the angular velocity in radians per unit of time. Since $\omega = \frac{\theta}{t}$, this formula can also be written as $v = r\omega$.

Example 1 **Determine the angular displacement in radians of 3.5 revolutions. Round to the nearest tenth.**

Each revolution equals 2π radians. For 3.5 revolutions, the number of radians is $3.5 \times 2\pi$, or 7π. 7π radians equals about 22.0 radians.

Example 2 **Determine the angular velocity if 8.2 revolutions are completed in 3 seconds. Round to the nearest tenth.**

The angular displacement is $8.2 \times 2\pi$, or 16.4π radians.

$\omega = \frac{\theta}{t}$

$\omega = \frac{16.4\pi}{3}$ *$\theta = 16.4\pi$, $t = 3$*

$\omega \approx 17.17403984$ *Use a calculator.*

The angular velocity is about 17.2 radians per second.

Example 3 **Determine the linear velocity of a point rotating at an angular velocity of 13π radians per second at a distance of 7 centimeters from the center of the rotating object. Round to the nearest tenth.**

$v = r\omega$

$v = 7(13\pi)$ *$r = 7$, $\omega = 13\pi$*

$v \approx 285.8849315$ *Use a calculator.*

The linear velocity is about 285.9 centimeters per second.

NAME ______________________ DATE ______________ PERIOD ________

Practice

Linear and Angular Velocity

Determine each angular displacement in radians. Round to the nearest tenth.

1. 6 revolutions

2. 4.3 revolutions

3. 85 revolutions

4. 11.5 revolutions

5. 7.7 revolutions

6. 17.8 revolutions

Determine each angular velocity. Round to the nearest tenth.

7. 2.6 revolutions in 6 seconds

8. 7.9 revolutions in 11 seconds

9. 118.3 revolutions in 19 minutes

10. 5.5 revolutions in 4 minutes

11. 22.4 revolutions in 15 seconds

12. 14 revolutions in 2 minutes

Determine the linear velocity of a point rotating at the given angular velocity at a distance r from the center of the rotating object. Round to the nearest tenth.

13. $\omega = 14.3$ radians per second, $r = 7$ centimeters

14. $\omega = 28$ radians per second, $r = 2$ feet

15. $\omega = 5.4\pi$ radians per minute, $r = 1.3$ meters

16. $\omega = 41.7\pi$ radians per second, $r = 18$ inches

17. $\omega = 234$ radians per minute, $r = 31$ inches

18. ***Clocks*** Suppose the second hand on a clock is 3 inches long. Find the linear velocity of the tip of the second hand.

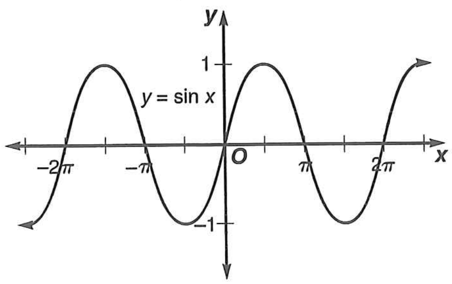
NAME ______________________ DATE ______________ PERIOD ________

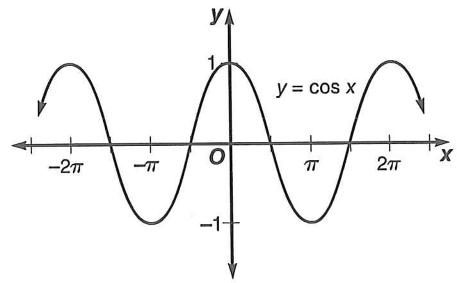
6-3

Study Guide

Graphing Sine and Cosine Functions

If the values of a function are the same for each given interval of the domain, the function is said to be **periodic.** Consider the graphs of $y = \sin x$ and $y = \cos x$ shown below. Notice that for both graphs the period is 2π and the range is from -1 to 1, inclusive.





Properties of the Graph of $y = \sin x$	Properties of the Graph of $y = \cos x$
The x-intercepts are located at πn, where n is an integer.	The x-intercepts are located at $\frac{\pi}{2} + \pi n$, where n is an integer.
The y-intercept is 0.	The y-intercept is 1.
The maximum values are $y = 1$ and occur when $x = \frac{\pi}{2} + 2\pi n$, when n is an integer.	The maximum values are $y = 1$ and occur when $x = \pi n$, where n is an even integer.
The minimum values are $y = -1$ and occur when $x = \frac{3\pi}{2} + 2\pi n$, where n is an integer.	The minimum values are $y = -1$ and occur when $x = \pi n$, where n is an odd integer.

Example 1 **Find $\sin \frac{7\pi}{2}$ by referring to the graph of the sine function.**

The period of the sine function is 2π. Since $\frac{7\pi}{2} > 2\pi$, rewrite $\frac{7\pi}{2}$ as a sum involving 2π.

$\frac{7\pi}{2} = 2\pi(1) + \frac{3\pi}{2}$ *This is a form of* $\frac{3\pi}{2} + 2\pi n$.

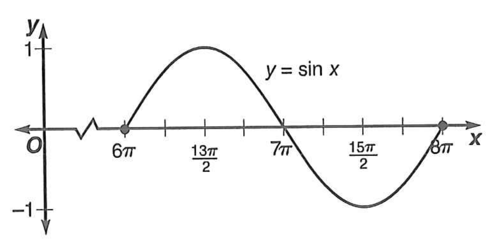
So, $\sin \frac{7\pi}{2} = \sin \frac{3\pi}{2}$ or -1.

Example 2 **Find the values of θ for which $\cos \theta = 0$ is true.**

Since $\cos \theta = 0$ indicates the x-intercepts of the cosine function, $\cos \theta = 0$ if $\theta = \frac{\pi}{2} + \pi n$, where n is an integer.

Example 3 **Graph $y = \sin x$ for $6\pi \leq x \leq 8\pi$.**

The graph crosses the x-axis at 6π, 7π, and 8π. Its maximum value of 1 is at $x = \frac{13\pi}{2}$, and its minimum value of -1 is at $x = \frac{15\pi}{2}$. Use this information to sketch the graph.



NAME ______________________ DATE ____________ PERIOD ________

6-3

Practice

Graphing Sine and Cosine Functions

Find each value by referring to the graph of the sine or the cosine function.

1. $\cos \pi$

2. $\sin \frac{3\pi}{2}$

3. $\sin \left(-\frac{7\pi}{2}\right)$

Find the values of θ for which each equation is true.

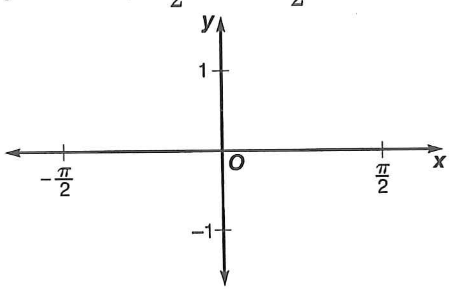
4. $\sin \theta = 0$

5. $\cos \theta = 1$

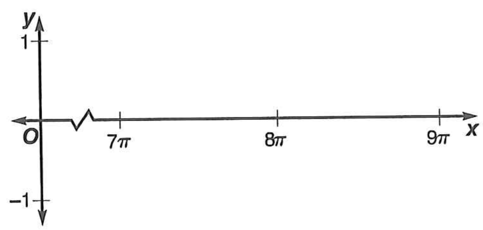
6. $\cos \theta = -1$

Graph each function for the given interval.

7. $y = \sin x;\ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

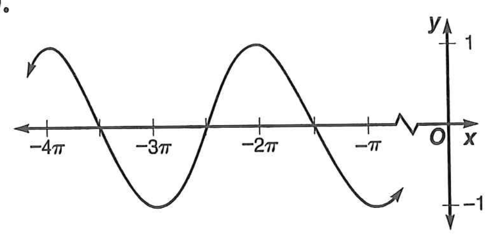


8. $y = \cos x;\ 7\pi \leq x \leq 9\pi$

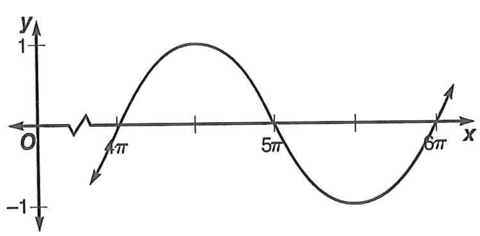


Determine whether each graph is y = sin x, y = cos x, or neither.

9.



10.



11. ***Meteorology*** The equation $y = 70.5 + 19.5 \sin\left[\frac{\pi}{6}(t - 4)\right]$ models the average monthly temperature for Phoenix, Arizona, in degrees Fahrenheit. In this equation, t denotes the number of months, with $t = 1$ representing January. What is the average monthly temperature for July?

NAME ______________________ DATE ______________ PERIOD ________

Study Guide

Amplitude and Period of Sine and Cosine Functions

The **amplitude** of the functions $y = A \sin \theta$ and $y = A \cos \theta$ is the absolute value of A, or $|A|$. The period of the functions $y = \sin k\theta$ and $y = \cos k\theta$ is $\frac{2\pi}{k}$, where $k > 0$.

Example 1 **State the amplitude and period for the function $y = -2 \cos \frac{\theta}{4}$.**

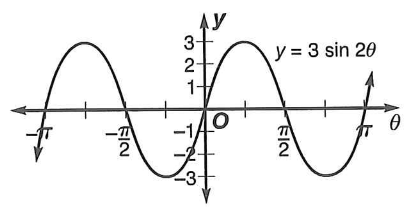
The definition of *amplitude* states that the amplitude of $y = A \cos \theta$ is $|A|$. Therefore, the amplitude of $y = -2 \cos \frac{\theta}{4}$ is $|-2|$, or 2.

The definition of *period* states that the period of $y = \cos k\theta$ is $\frac{2\pi}{k}$. Since $-2 \cos \frac{\theta}{4}$ equals $-2 \cos \left(\frac{1}{4}\theta\right)$, the period is $\frac{1}{4}$ or 8π.

Example 2 **State the amplitude and period for the function $y = 3 \sin 2\theta$. Then graph the function.**

Since $A = 3$, the amplitude is $|3|$ or 3.
Since $k = 2$, the period is $\frac{2\pi}{2}$ or π.

Use the amplitude and period above and the basic shape of the sine function to graph the equation.



Example 3 **Write an equation of the sine function with amplitude 6.7 and period 3π.**

The form of the equation will be $y = A \sin k\theta$. First find the possible values of A for an amplitude of 6.7.

$|A| = 6.7$
$A = 6.7$ or -6.7

Since there are two values of A, two possible equations exist.

Now find the value of k when the period is 3π.

$\frac{2\pi}{k} = 3\pi$ *The period of the sine function is $\frac{2\pi}{k}$.*

$k = \frac{2\pi}{3\pi}$ or $\frac{2}{3}$

The possible equations are $y = 6.7 \sin \frac{2}{3}\theta$ or $y = -6.7 \sin \frac{2}{3}\theta$.

NAME ____________________ DATE ______________ PERIOD _________

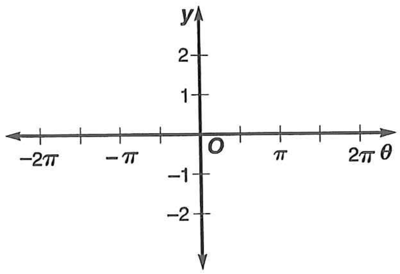
6-4

Practice

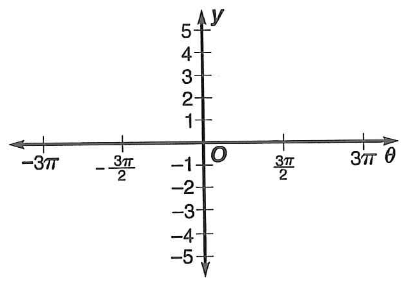
Amplitude and Period of Sine and Cosine Functions

State the amplitude and period for each function. Then graph each function.

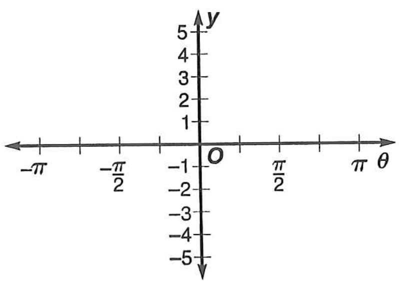
1. $y = -2 \sin \theta$



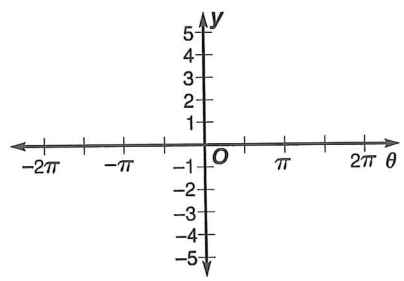
2. $y = 4 \cos \frac{\theta}{3}$



3. $y = 1.5 \cos 4\theta$



4. $y = -\frac{2}{3} \sin \frac{\theta}{2}$



Write an equation of the sine function with each amplitude and period.

5. amplitude = 3, period = 2π

6. amplitude = 8.5, period = 6π

Write an equation of the cosine function with each amplitude and period.

7. amplitude = 0.5, period = 0.2π

8. amplitude = $\frac{1}{5}$, period = $\frac{2}{5}\pi$

9. ***Music*** A piano tuner strikes a tuning fork for note A above middle C and sets in motion vibrations that can be modeled by the equation $y = 0.001 \sin 880\pi t$. Find the amplitude and period for the function.

NAME ______________________ DATE ____________ PERIOD ________

6-5

Study Guide

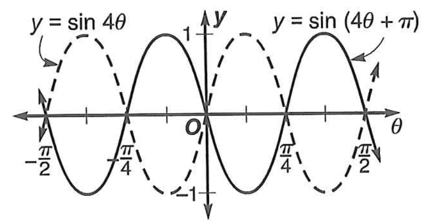
Translations of Sine and Cosine Functions

A horizontal translation of a trigonometric function is called a **phase shift**. The phase shift of the functions $y = A \sin (k\theta + c)$ and $y = A \cos (k\theta + c)$ is $-\frac{c}{k}$, where $k > 0$. If $c > 0$, the shift is to the left. If $c < 0$, the shift is to the right. The **vertical shift** of the functions $y = A \sin (k\theta + c) + h$ and $y = A \cos (k\theta + c) + h$ is h. If $h > 0$, the shift is upward. If $h < 0$, the shift is downward. The **midline** about which the graph oscillates is $y = h$.

Example 1 **State the phase shift for $y = \sin (4\theta + \pi)$. Then graph the function.**

The phase shift of the function is $-\frac{c}{k}$ or $-\frac{\pi}{4}$.

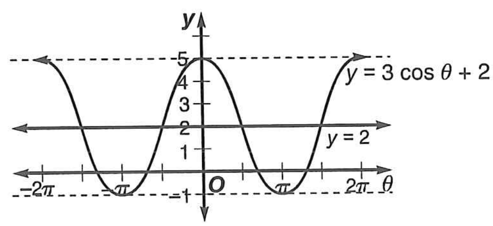
To graph $y = \sin (4\theta + \pi)$, consider the graph of $y = \sin 4\theta$. The graph of $y = \sin 4\theta$ has an amplitude of 1 and a period of $\frac{\pi}{2}$. Graph this function, then shift the graph $-\frac{\pi}{4}$.



Example 2 **State the vertical shift and the equation of the midline for $y = 3 \cos \theta + 2$. Then graph the function.**

The vertical shift is 2 units upward. The midline is the graph of $y = 2$.

To graph the function, draw the midline. Since the amplitude of the function is $|3|$, or 3, draw dashed lines parallel to the midline which are 3 units above and below $y = 2$. That is, $y = 5$ and $y = -1$. Then draw the cosine curve with a period of 2π.



Example 3 **Write an equation of the cosine function with amplitude 2.9, period $\frac{2\pi}{5}$, phase shift $-\frac{\pi}{2}$, and vertical shift -3.**

The form of the equation will be $y = A \cos (k\theta + c) + h$. Find the values of A, k, c, and h.

A: $|A| = 2.9$
$A = 2.9$ or -2.9

k: $\frac{2\pi}{k} = \frac{2\pi}{5}$ *The period is* $\frac{2\pi}{5}$.
$k = 5$

c: $-\frac{c}{k} = -\frac{\pi}{2}$ *The phase shift is* $-\frac{\pi}{2}$.
$-\frac{c}{5} = -\frac{\pi}{2}$ $k = 5$
$c = \frac{5\pi}{2}$

h: $h = -3$

The possible equations are $y = \pm 2.9 \cos \left(5\theta + \frac{5\pi}{2}\right) - 3$.

NAME ______________________ DATE ______________ PERIOD ________

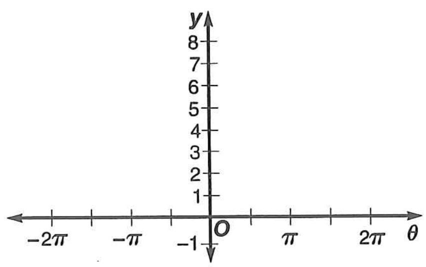
6-5

Practice

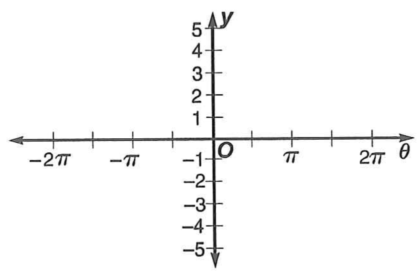
Translations of Sine and Cosine Functions

State the vertical shift and the equation of the midline for each function. Then graph each function.

1. $y = 4 \cos \theta + 4$

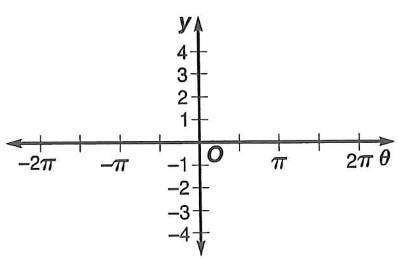


2. $y = \sin 2\theta - 2$

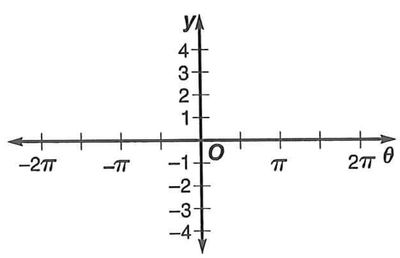


State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

3. $y = 2 \sin\left(\theta + \frac{\pi}{2}\right) - 3$



4. $y = \frac{1}{2} \cos (2\theta - \pi) + 2$



Write an equation of the specified function with each amplitude, period, phase shift, and vertical shift.

5. sine function: amplitude = 15, period = 4π, phase shift = $\frac{\pi}{2}$, vertical shift = -10

6. cosine function: amplitude = $\frac{2}{3}$, period = $\frac{\pi}{3}$, phase shift = $-\frac{\pi}{3}$, vertical shift = 5

7. sine function: amplitude = 6, period = π, phase shift = 0, vertical shift = $-\frac{3}{2}$

Study Guide

Modeling Real-World Data with Sinusoidal Functions

Example **The table shows the average monthly temperatures for Ann Arbor, Michigan. Write a sinusoidal function that models the average monthly temperatures, using $t = 1$ to represent January. Temperatures are in degrees Fahrenheit (°F).**

Jan.	30°
Feb.	34°
Mar.	45°
Apr.	59°
May	71°
June	80°
July	84°
Aug.	81°
Sept.	74°
Oct.	62°
Nov.	48°
Dec.	35°

These data can be modeled by a function of the form $y = A \sin(kt + c) + h$, where t is the time in months.

First, find A, h, and k.

A: $A = \frac{84 - 30}{2}$ or 27 — *A is half the difference between the greatest temperature and the least temperature.*

h: $h = \frac{84 + 30}{2}$ or 57 — *h is half the sum of the greatest value and the least value.*

k: $\frac{2\pi}{k} = 12$ — *The period is 12.*

$k = \frac{\pi}{6}$

Substitute these values into the general form of the function.

$y = A \sin(kt + c) + h$ $\qquad$ $y = 27 \sin\left(\frac{\pi}{6}t + c\right) + 57$

To compute c, substitute one of the coordinate pairs into the equation.

$y = 27 \sin\left(\frac{\pi}{6}t + c\right) + 57$	
$30 = 27 \sin\left[\frac{\pi}{6}(1) + c\right] + 57$	*Use (t, y) = (1, 30).*
$-27 = 27 \sin\left(\frac{\pi}{6} + c\right)$	*Subtract 57 from each side.*
$-\frac{27}{27} = \sin\left(\frac{\pi}{6} + c\right)$	*Divide each side by 27.*
$\sin^{-1}(-1) = \frac{\pi}{6} + c$	*Definition of inverse*
$\sin^{-1}(-1) - \frac{\pi}{6} = c$	*Subtract $\frac{\pi}{6}$ from each side.*
$-2.094395102 \approx c$	*Use a calculator.*

The function $y = 27 \sin\left(\frac{\pi}{6}t - 2.09\right) + 57$ is one model for the average monthly temperature in Ann Arbor, Michigan.

NAME ______________________ DATE ____________ PERIOD ________

6-6

Practice

Modeling Real-World Data with Sinusoidal Functions

1. ***Meteorology*** The average monthly temperatures in degrees Fahrenheit (°F) for Baltimore, Maryland, are given below.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
32°	35°	44°	53°	63°	73°	77°	76°	69°	57°	47°	37°

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.

b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.

c. What is the period of a sinusoidal function that models the monthly temperatures?

d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.

e. According to your model, what is the average temperature in July? How does this compare with the actual average?

f. According to your model, what is the average temperature in December? How does this compare with the actual average?

2. ***Boating*** A buoy, bobbing up and down in the water as waves move past it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 3 feet.

a. What is the amplitude of a sinusoidal function that models the bobbing buoy?

b. What is the period of a sinusoidal function that models the bobbing buoy?

c. Write a sinusoidal function that models the bobbing buoy, using $t = 0$ at its highest point.

d. According to your model, what is the height of the buoy at $t = 2$ seconds?

e. According to your model, what is the height of the buoy at $t = 6$ seconds?

