

The background features a vibrant circus theme. At the top, three red and white striped tents with red flags on their peaks are set against a sky of radiating red and blue stripes. Below the tents is a large, dark green archway with a scalloped edge, decorated with small white stars. The archway frames a deep red curtain that has a subtle, repeating pattern of the same stars. A blue banner with a white border is positioned across the middle of the curtain, containing the text '4.5 Linear Programming'.

4.5 Linear Programming

Goals

- ▶ Write inequalities for business constraints
- ▶ Graph inequalities
- ▶ Find points of intersection
- ▶ Test points to maximize/minimize profits



Profit

- ▶ Inventory – goods left on hand
- ▶ **Efficient**
 - ▶ Working well
 - ▶ Producing a product with a minimum amount of energy, expense, and waste



Constraints

- ▶ **Constraints are :**
 - ▶ Conditions that must be met by a business
 - ▶ (written as linear inequalities)
 - ▶ **Examples:**
 - Money available for investment
 - Time available
 - Materials available
 - Demand for product or service



Constraints

- ▶ **Linear Programming:**

- ▶ a method for planning within given constraints

- ▶ We use linear programming to maximize and minimize factors in a business situation.

- ▶ Maximize = find the greatest value within constraints

- ▶ Minimize = find the least value within the constraints

- ▶ Example:

- ▶ Businesses wish to maximize profit and minimize cost



Skill 1

- ▶ Joe and Brett are planning a road trip to the World Series.
 - ▶ A. They are leaving at 10:00 am and do not want to drive after dark. They only have 9 hrs to drive
 - ▶ B. They want to travel at least 400 miles.
 - ▶ C. They want to stay within the speed limit of 50 miles/hr
- ▶ We want to graph the constraints as inequalities and show the different possible ways to make the trip.



▶ **Assign variables**

▶ Let $x = \text{hours}$

$y = \text{miles}$

▶ **Write inequalities for each constraint**

▶ A. The trip is no more than 9 hrs

$$x \leq 9$$

$$x \geq 0$$

▶ B. The distance must be at least 400 miles

$$y \geq 400$$

▶ C. speed limit of 50 (Distance = rate * time)

$$y \leq 50x$$

▶ **Graph the system of inequalities**

▶

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- ▶ After we graph the inequalities we find the corner points of the shaded region by solving the equations that intersect at the given point.

- ▶ Point A

- $y = 400$

- $y = 50x$

- $50x = 400$

- $x = 8$

- $y = 400$

- A (8,400)

- What does this point represent?



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- ▶ After we graph the inequalities we find the corner points of the shaded region by solving the equations that intersect at the given point.

- ▶ Point B

- $y = 400$

- $x = 9$

- $B (9, 400)$

- What does this point represent?



▶ Point C

$$x = 9$$

$$y = 50x$$

$$y = 50x$$

$$y = 50(9)$$

$$y = 450$$

C (9,450)

▶ What does this point represent?



SKILL 2

- ▶ Nick, Pavel, and Henrick have begun selling bobble heads and souvenir pucks. They purchase the bobble heads for \$4.50 and the pucks for \$2.50. There are some constraints that affect their business.
 - ▶ To satisfy demand, they must produce at least 30 items per week.
 - ▶ The supplier can supply them no more than 20 pucks per week.
 - ▶ Because of free puck night at the Joe they must be ready to sell at least as many pucks as bobble heads.



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- ▶ Assign variables
 - ▶ Express the constraints as inequalities:
 - ▶ They must sell a total more than 30 items.
 - ▶ $x + y \geq 30$
 - In slope intercept : $y \geq -x + 30$
 - ▶ They can obtain no more than 20 pucks
 - ▶ $y \leq 20$
 - ▶ They will sell at least as many pucks as bobble heads.
 - ▶ $y \geq x$
 - ▶ Graph
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- ▶ We want to find the lowest costs within the given constraints.
 - ▶ We need to choose a quantity that we want to maximize or minimize and write an equation for that quantity. **OBJECTIVE FUNCTION**
 - ▶ Identify the constraints
 - ▶ We want to minimize total cost. Cost is expressed by the following equation:
 - $C = 4.50x + 2.50y$ where c = the cost
 - x = the number of bobble heads
 - y = the number of souvenir pucks
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- ▶

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- ▶ Substitute vertices in the objective function $c = 4.50x + 2.50y$.
 - ▶ Where is the lowest cost achieved?

