10-3 OWNING AND OPERATING COSTS

When Freda became a salesperson in the business started by her friends Evelyn and Greg, she began to use her car for business. She distributed posters and flyers and also delivered the finished T-shirts and sweatshirts.

Freda soon found that Hari, who kept the company's financial records, wanted an accurate report on the car expenses. So Freda began keeping all of her receipts for the purchase of gas and for car maintenance. Hari said that the business will at least pay for her gas. Freda also has monthly payments on her car loan and insurance premiums to pay. She asked Hari whether the company could pay part of these bills. Hari said he would speak to an accountant to find out more about personal and business expenses.

After learning more about business expenses for a car, Hari and Freda looked into car leasing. They found that in some circumstances it makes sense financially for a business to lease, rather than own, a company car.

OBJECTIVES: *In this lesson, we will help Freda to:*

- *Calculate car mileage per gallon of gasoline.*
- *Compute operating costs of a car and total costs per mile.*
- *Compute the costs of leasing a car.*

MILEAGE

Hari tells Freda that she should keep all receipts for the purchase of gas and also keep track of miles driven for business. Each time she uses the car for business, she is to write down the beginning and ending odometer readings. The **odometer** indicates the number of miles a car has been driven. Freda also is to calculate **miles per gallon,** the average number of miles her car travels on each gallon of gas.

MAINTENANCE

Since Freda's car has only 12,000 miles on it, expenses for tune-ups, parts, and repairs should be low. Her car is still under **warranty,** which means that the manufacturer pays for many repairs. The warranty expires after a certain number of miles or years.

To maintain her car in good condition, Freda has the oil changed every 3000 miles. She has a tune-up every 10,000 miles and regularly checks the air pressure in the tires and the levels of engine coolant.

REPAIRS

Freda has talked with mechanics and has learned that she will eventually need a new battery, new tires, and brake linings. She will also need to replace filters and hoses. She may need replacements for or repairs on a variety of other parts including the fuel pump, alternator, and muffler. Freda plans to set aside an amount of money each month so that she will be able to pay her bills for repairs as they come up.

LEASING A CAR

Because car expenses are often high and because cars depreciate, car leasing has become popular. **Leasing** is an arrangement by which a car is rented for a monthly fee under a contract that extends for several years. The person leasing the car pays for gas; the company that leases the car pays some maintenance and repair expenses.

ASK YOURSELF

1. What is an odometer?
2. What is the difference between maintenance and repairs?
3. What is meant by "miles per gallon?"
4. What does leasing mean?

ALGEBRA *REVIEW*

1. Find the average of 4, 7, and 9.
2. Jack walked 15 miles in 5 hours. What was his average speed in miles per hour?
3. Pat ran 8 miles in 70 minutes. Find her average rate in minutes per mile.
4. Ms. Gonzales drove 140 miles in 3.5 hours. Find her average speed in miles per hour.
5. Mr. Oliver used 12 gallons of gas to drive 156 miles. What was his average miles per gallon?

Find the solution of each system of equations.

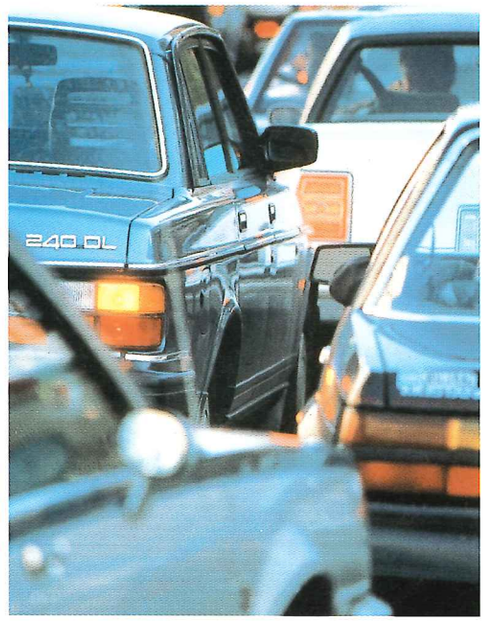
6. $x + y = 10$
   $y = 5$
7. $y = 2x + 1$
   $y = 15$
8. $y = 2x - 3$
   $x = 2.5$

SHARPEN YOUR SKILLS

SKILL 1

The number of miles that you can drive for each gallon of gas depends on the kind of driving you do. Steady driving at a moderate speed gives the best miles per gallon. Idling or moving at very low speeds in city traffic and driving at high speeds use more gas and lower the miles per gallon rate.

Miles per gallon of gas is like miles per hour except that you divide total miles by the number of gallons of gas used rather than by the hours driven.



Average Miles per Gallon

$a = m \div g$ where a = average miles per gallon
m = total miles driven
g = total gallons purchased

Freda learned from a friend the way to determine the average miles per gallon of gas.

1. Record the mileage when the gas tank is full.
2. Fill the tank the next time you purchase gas; then record the mileage, gallons purchased, and amount spent.
3. Follow this procedure several times so that you have enough miles and gallons for a valid average.

EXAMPLE 1 Freda filled her gas tank four times, recording the following information. She did not record the gallons or dollar amount the first time because she did not have a previous odometer reading and therefore could not calculate the mileage she had traveled.

Odometer Reading	Gallons of Gas	Amount
11,400	——	——
11,604	9.7	$12.03
11,842	11.2	13.89
12,027	8.6	10.67

QUESTION For the given information, what is the average miles per gallon?

SOLUTION

Freda can find the answer by using the formula $a = m \div g$.

Total miles driven: $m = 12{,}027 - 11{,}400 = 627$
Total gallons purchased: $g = 9.7 + 11.2 + 8.6 = 29.5$
Average miles per gallon: $a = 627 \div 29.5 = 21.3$

Freda averages 21.3 miles per gallon of gas.

EXAMPLE 2 Freda has purchased gas three times to drive 627 miles in Example 1.

QUESTION What is Freda's cost per mile for gas?

SOLUTION

The cost per mile, like gallons per mile, is an average. It is found by dividing total cost by the number of miles. Since we have used a for average miles per gallon, we will use b for the cost per mile.

Average Cost per Mile

$b = c \div m$ where b = average cost per mile
c = total cost
m = total miles driven

According to Freda's records,

Total cost:	\$12.03 + \$13.89 + \$10.67 = \$36.59
Total miles driven:	627
Average cost per mile:	$b = 36.59 \div 627 = 0.058$

The average cost per mile for gas is \$0.058 or 5.8 cents. This does not seem like a lot of money, but, as we shall see, the total cost per mile of owning and operating a car is much higher.

SKILL 2

Freda is driving about 1000 miles per month. Using this fact and the information from Example 2, she calculates that her yearly expense for gas will be $12 \cdot 1000 \cdot 0.058 = \$696$.

In addition to the purchase of gas, Freda has to pay the following.

- Regular maintenance: tune-up, oil change, filters, wheel alignment. This will cost about \$125 per year.
- Repairs: Some can be anticipated, such as new tires, but most cannot be known in advance. Freda can be reasonably sure of some repair expenses, and she knows that these will increase with time. Freda feels that \$240 per year will be a safe estimate for now.
- Monthly loan payment: \$304.72
- Insurance premium: \$428.60, four times per year
- Taxes and fees: \$94 per year

EXAMPLE 3 Using the information given, Freda decides to set up a budget.

QUESTION How should Freda set up a budget for her car expenses?

SOLUTION

All expenses must be annualized, that is, the total amount per year must be calculated for each expense. Gasoline, maintenance, repairs, and taxes/fees are already stated as annual. The loan payment is monthly. Insurance is paid four times a year. We multiply to annualize these expenses.

Loan payment: $12(304.72) = 3656.64$
Insurance: $4(428.60) = 1714.40$

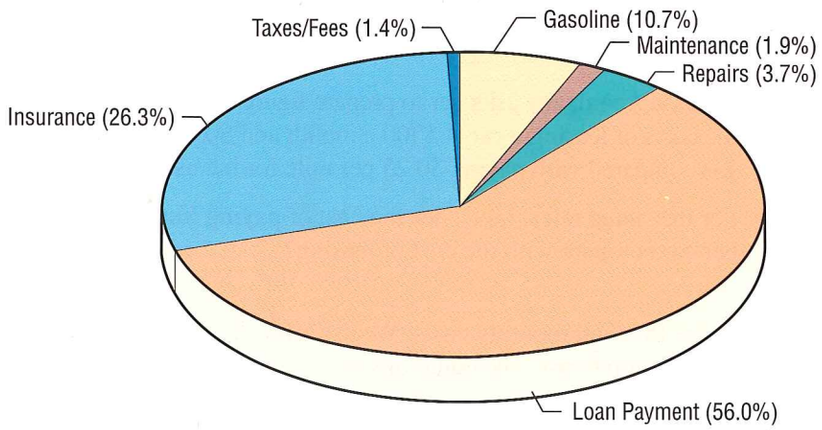
Annual expenses are shown in the following table.

Annual Expenses (Dollars)					
Gasoline	**Maintenance**	**Repairs**	**Loan Payment**	**Insurance**	**Taxes/Fees**
696	125	240	3656.64	1714.40	94

After looking at the yearly budget, Freda decides that she needs a monthly budget to know how much money to put aside for monthly bills each month and to get a better idea of what the car is costing her. She divides each annual amount by 12 to find the monthly amounts.

Monthly Expenses (Dollars)					
Gasoline	**Maintenance**	**Repairs**	**Loan Payment**	**Insurance**	**Taxes/Fees**
58	10.42	20	304.72	142.87	7.83

Freda was surprised when she saw what her car was costing her each month. She hadn't realized how much it was because she pays the bills at so many different times. She used her spreadsheet to make a pie graph of the costs.



EXAMPLE 4 Freda totaled her monthly expenses and found that they came to $543.84.

QUESTION What is Freda's average cost per mile to own and operate her car?

SOLUTION

Freda drives an average of 1000 miles per month, so the average cost per mile can be found by dividing her total costs by 1000.

$$543.84 \div 1000 = 0.544$$

Freda can't believe that it costs her more than 50 cents for every mile that she drives. She shows her figures to Hari. He has been investigating car costs and tells her that the business should pay some of her operating expenses but not the loan payment. He says that she is buying the car, so it will belong to her, not to the business.

Freda then calculates her car expenses, leaving out the loan payment.

Expenses:	$58 + 10.42 + 20 + 142.87 + 7.83 = 239.12$
Average per mile:	$239.12 \div 1000 = 0.239$

Freda's operating expenses are $0.239 or 23.9 cents per mile.

Hari and the others agree that it is fair to pay Freda 24 cents a mile for the business use of her car. They later discover that this is about the amount that most employees are allowed to charge when using their cars for business.

SKILL 3

Freda's cousin, Vanessa, drives for a delivery company. At first the company pays her 25 cents a mile. Then, as business increases, the company considers leasing a car. It can lease a car for $300 a month. The leasing agency pays for all expenses (except gas) and repairs. Leasing would save wear and tear on Vanessa's car, but would it save money for the company?

EXAMPLE 5 Vanessa's company asks her to prepare figures and graphs showing the costs of leasing a car at $300 a month and $0.058 per mile for gas compared with paying $0.25 per mile for the use of her car.

QUESTION For the stated rates, how does the cost of paying Vanessa to use her car compare with the cost of leasing?

SOLUTION

First we compare the total monthly costs. We use x for the number of miles driven and y for the total cost. The equations are

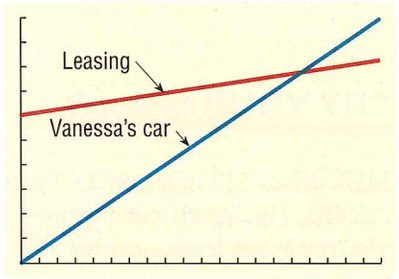
$y = 0.25x$	Cost of using Vanessa's car
$y = 300 + 0.058x$	Cost of leasing plus 0.058 per mile
$0.25x = 300 + 0.058x$	Substitute $0.25x$ for y in the second equation.
$0.192x = 300$	Combine x terms.
$x = 1562.5$	

The costs are equal at 1562.5 miles per month. Since some rounding is involved in these calculations, you can say that if Vanessa drives more than 1560 miles per month, the company will save by leasing a car. Graphing will give a better idea of the comparison.

Using a graphing calculator, you can graph these two equations as shown.

Range:

Xmin:	0	Ymin:	0
Xmax:	2000	Ymax:	500
Xscl:	100	Yscl:	50



EXAMPLE 6 Vanessa's company would like to compare the cost per mile of leasing with paying \$0.25 for using Vanessa's car.

QUESTION How do the costs per mile compare?

SOLUTION

In this case the cost of using Vanessa's car is fixed at 25 cents a mile. To find the cost per mile of leasing c, you must divide the monthly leasing cost by the number of miles x, and then add the cost per mile for gas.

The cost per mile equations are

$c = 0.25$ Cost per mile of using Vanessa's car

$c = \frac{300}{x} + 0.058$ Cost per mile of leasing a car

Solve these equations for x.

$0.25 = \frac{300}{x} + 0.058$ Substitute 0.25 for c in the second equation.

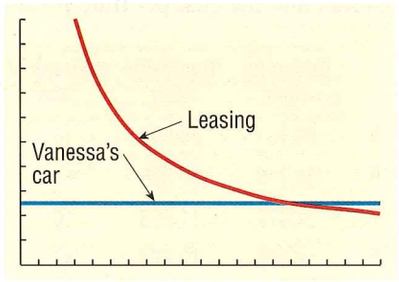
$0.192 = \frac{300}{x}$ Subtract 0.058 from both sides.

$0.192x = 300$

$x = 1562.5$

Graph the cost per mile equations on a graphing calculator using the following range values. Use X and Y for the variables x and c in the equations.

Xmin:	0	Ymin:	0
Xmax:	2000	Ymax:	1
Xscl:	100	Yscl:	0.1



The tracing feature on the calculator can be used to find the cost per mile for different total monthly amounts. For example, the cost of leasing at 1000 miles per month is 35.8 cents per mile, and the cost at 1600 miles per month is 24.6 cents per mile.

TRY YOUR SKILLS

Mark drives 537 miles on 23.7 gallons of gas. He pays $30.57 for the gas. He calculates his yearly car expenses, including loan payments of $2560, to be $5470. He can lease a car for $300 per month.

1. Find Mark's average miles per gallon.
2. Find his average cost per gallon.
3. Find his average monthly expense.
4. Find Mark's average cost per mile if he drives 12,000 miles per year.
5. Find the cost of leasing for a month and driving 1350 miles while paying an average of 5.7 cents per mile for gas.
6. Write equations for the cost of paying Mark a per mile amount for operating expenses (not counting the loan) and for the leasing cost.

EXERCISE YOUR SKILLS

KEY TERMS

leasing
miles per gallon
odometer
warranty

1. Car companies advertise the miles per gallon that can be expected from a car. Why should this be considered when buying a car?
2. Why is it important to take care of minor repairs promptly?
3. Why should a car owner estimate repair expenses when preparing an annual budget?
4. What are some of the differences between owning and leasing a car?

Use the information in the table to find the distance traveled, the miles per gallon, and the cost per mile for the trip.

	Odometer First	Odometer Last	Gallons of Gas	Cost of Gas	Distance	Miles per Gallon	Cost per Mile
5.	28,431	28,848	18.4	$25.15			
6.	38,715	39,326	15.7	18.95			
7.	11,477	11,628	10.1	14.00			
8.	18,388	19,374	40.6	63.74			
9.	15,428	15,639	10.9	14.38			

Isabel drives 15,000 miles a year and has the following real and estimated car costs:

Maintenance: $150 per year
Repairs: $300 per year
Loan payments: $401.30 per month
Insurance: $603.27 twice per year
Taxes: $114 per year
Gasoline: $0.06 per mile

Find each of the following for Isabel.

10. Her annual expense for gasoline

11. Her monthly expense for insurance

12. The total of her annual car expenses

13. Her average cost per mile to own and operate her car

14. Her annual cost to operate the car not including the monthly loan payment

15. Create a pie graph to show her annual expenses for her car.

Mark works for a company that will make a decision about whether to pay him 20 cents per mile to drive his car for the business or to lease a car for $200 per month and 6 cents per mile.

16. Write equations for the total monthly cost under each arrangement.

17. Solve the equations simultaneously to find the number of miles per month for which the costs will be equal.

18. Graph the two equations to show the relationship.

19. Which is the better arrangement for 1000 miles per month?

20. What is the difference in costs for a 1000-mile month?

21. Which is the better arrangement for 2000 miles per month?

22. What is the difference in costs for a 2000-mile month?

MIXED REVIEW

1. Jacob borrowed $10,000 and repaid it at the rate of $200 per month for 5 years. Find the total interest paid over the 5 years.

Phyllis bought 1200 shares of stock at $37\frac{3}{8}$ per share. In Exercises 2–4, ignore the commission costs.

2. Find the cost of the shares.

3. After 2 years, Phyllis sold the shares at $45\frac{5}{8}$ per share. What was the total sale price?

4. What was the capital gain or loss on the shares?

5. Use IRS Form 1040EZ to find the amount of the refund or payment due to the nearest dollar for a single taxpayer who had $1,600 withheld on his $13,900 income. The taxpayer has no taxable interest and cannot be claimed as a dependent by someone else.

10-4 INSURANCE COSTS

Trevor, who is 17, is allowed to drive the family car. Sometimes, he drives his 11-year-old sister Tracey and her friends to a weekend event. Last Saturday, Trevor drove Tracey and her friends, Roy and Leah, to a movie. As he drove, another car came speeding past, rapidly changing lanes and weaving back and forth among the other cars. Just as the car passed, Trevor was distracted for an instant by the younger children in the back seat, who were wrestling over a piece of candy.

As Trevor looked up, the car that was speeding collided with another car. Both cars seemed to be heavily damaged as they skidded off the road. Trevor had to change lanes quickly and slow down to avoid another car that had moved into his lane because of the accident. His sister and her friends had their seat belts on, but they were shaken up.

As a result of seeing the accident, Trevor asked his parents to tell him about insurance. He wanted to know what it is, why it is needed, and why costs are higher for young people. His parents explained that by collecting money from thousands of car owners, the insurance company can pay for car repairs and medical bills after accidents.

Compared with the many thousands of people who purchase insurance, only a few actually have accidents. But no one knows which ones will have the accidents, so everyone should purchase insurance. Furthermore, in most states, the law requires that you have auto insurance.

OBJECTIVES: *In this lesson, we will help Trevor to:*

- *Examine several types of insurance that are available for car owners.*
- *Compare rates of auto insurance for different drivers on the basis of the age, sex, training, and marital status of the driver and several other factors.*
- *Understand how insurance companies use accident statistics to estimate their costs.*