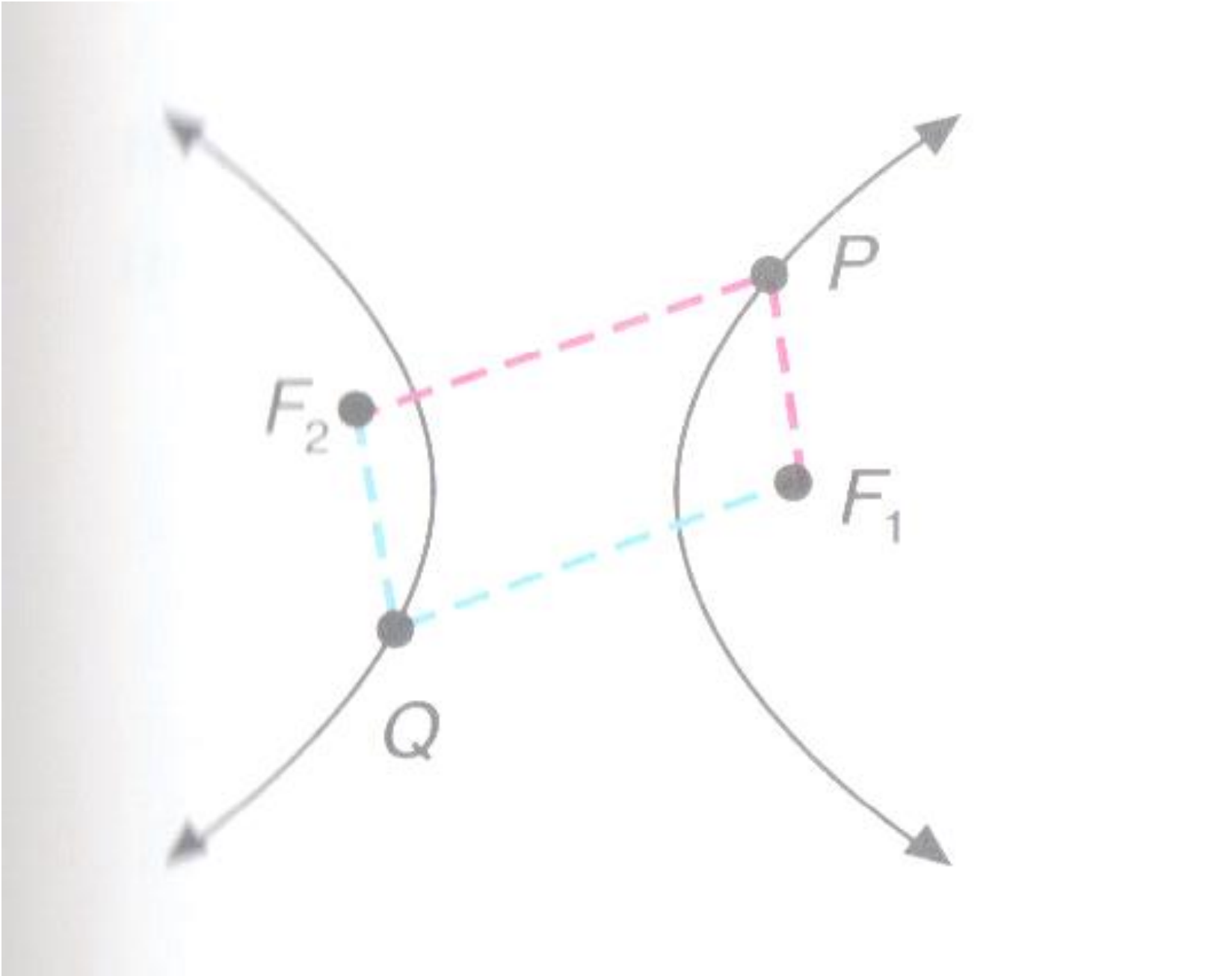
Hyperbolas

Goals

- Use the standard and general forms of the equation of a hyperbola
- Graph

Definitions

- Hyperbola-is the locus of all points in the plane such that the absolute value of the differences of the distance from two “given points” in the plane is constant.


P
F_2
F_1
Q

Definitions

- Foci-the “given points”
- If F_1 and F_2 are the foci and *P* and *Q* are any two points on the hyperbola, $\left|PF_1 - PF_2\right| = \left|QF_1 - QF_2\right|$

Definitions

- The center of a hyperbola is the midpoint of the segments whose endpoints are the foci.
- The point on each branch of the hyperbola nearest the center is called the vertex.

Definitions

- Asymptotes-are the lines that the curve approaches, but never touches as the relation increases or decreases toward ∞.

Definitions

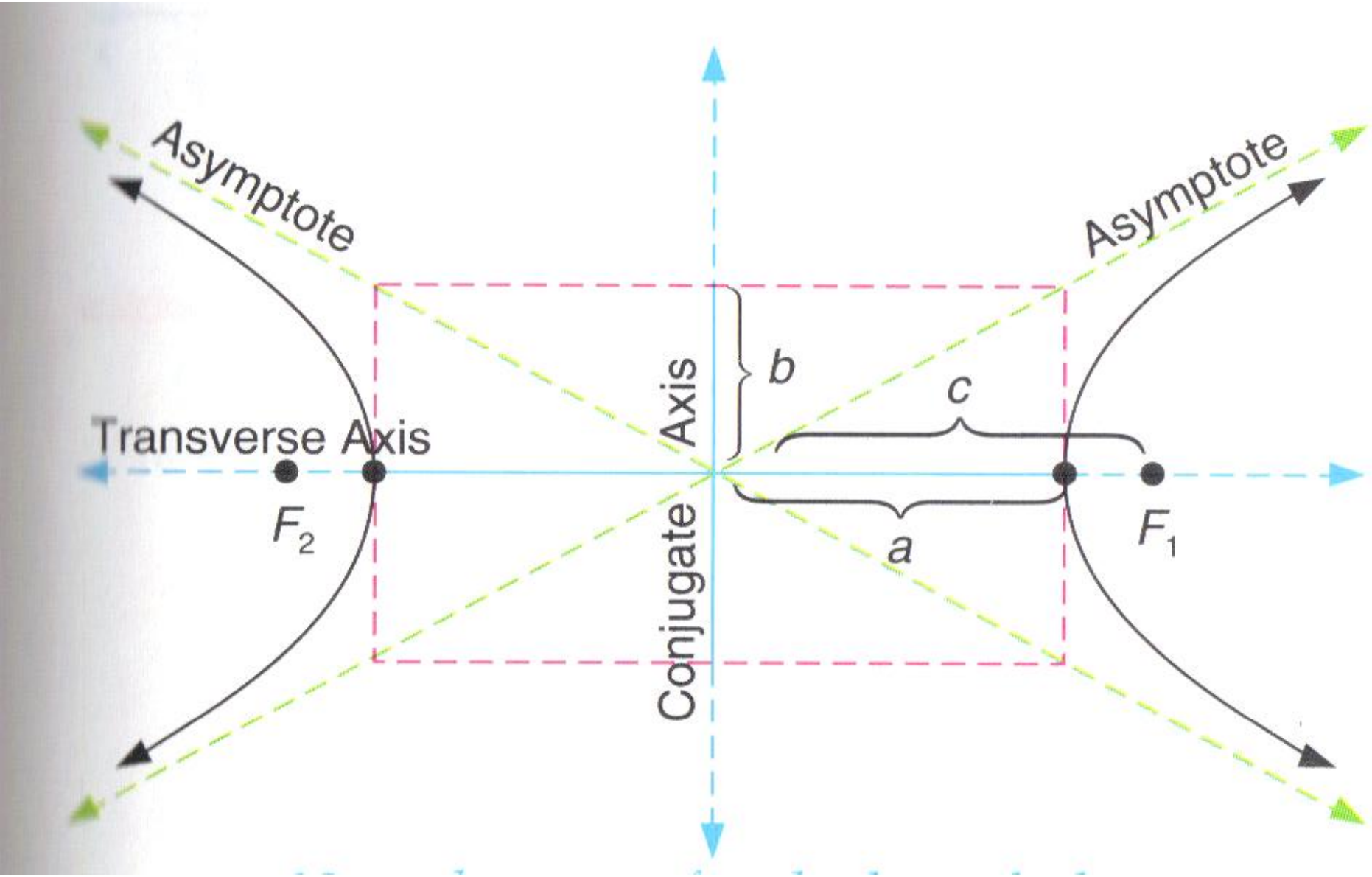
- The distance from the vertex to the center is *a* units
- The distance from the foci to the center is *c* units

Definitions

- A hyperbola has two axes of symmetry. The line with the endpoints at the vertices is the transverse axis, and has length ___?

Definitions

- The line segment ⊥ to the transverse axis and thru the center is the conjugate axis, and has length _____?

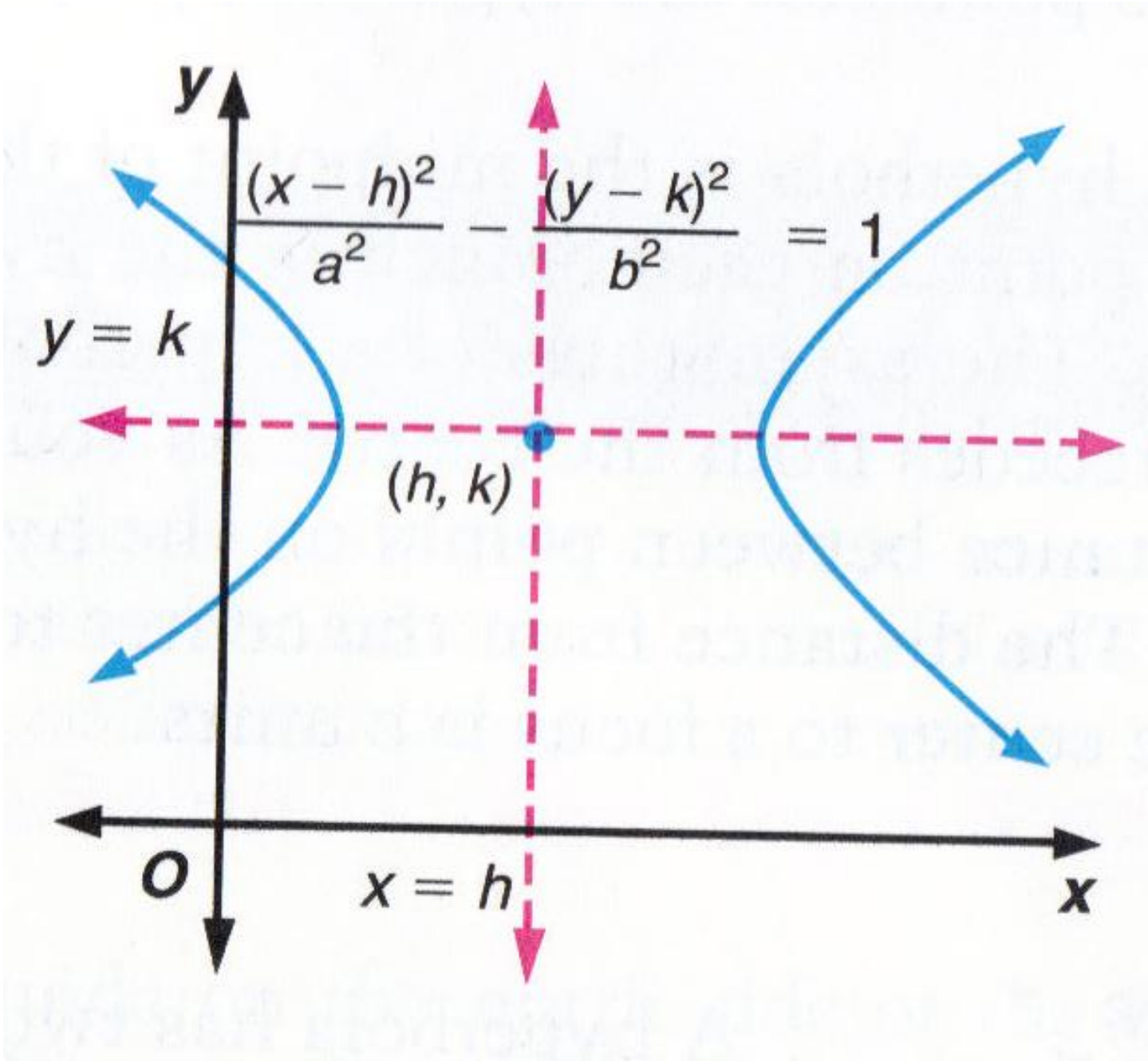

Asymptote
Asymptote
Transverse Axis
Conjugate Axis
b
c
a
F_2
F_1

Standard Form

- With center at (h,k) and transverse axis of length 2a, where $b^2 = c^2 - a^2$ is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

- When the transverse axis is ll to the x axis

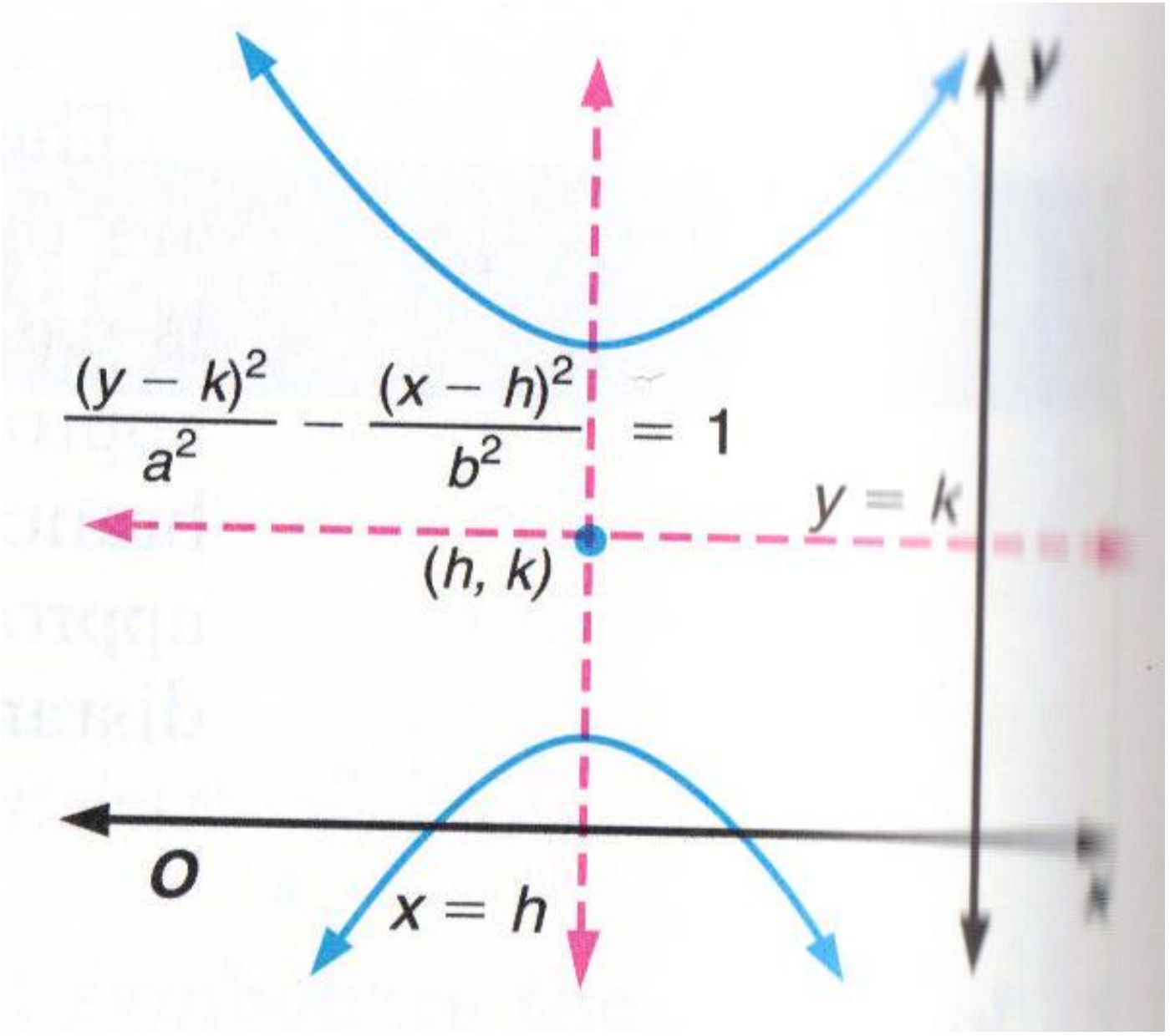

y
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
y = k
(h, k)
O
x = h
x

Standard Form

- With center at (h,k) and transverse axis of length 2a, where $b^2 = c^2 - a^2$ is

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

- When the transverse axis is ll to the y axis


$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
y
y = k
(h, k)
O
x = h
x

Example 1

- Find the equation of a hyperbola if the foci are at (4,0) and (-4,0) and the vertices are at (1,0) and (-1,0).

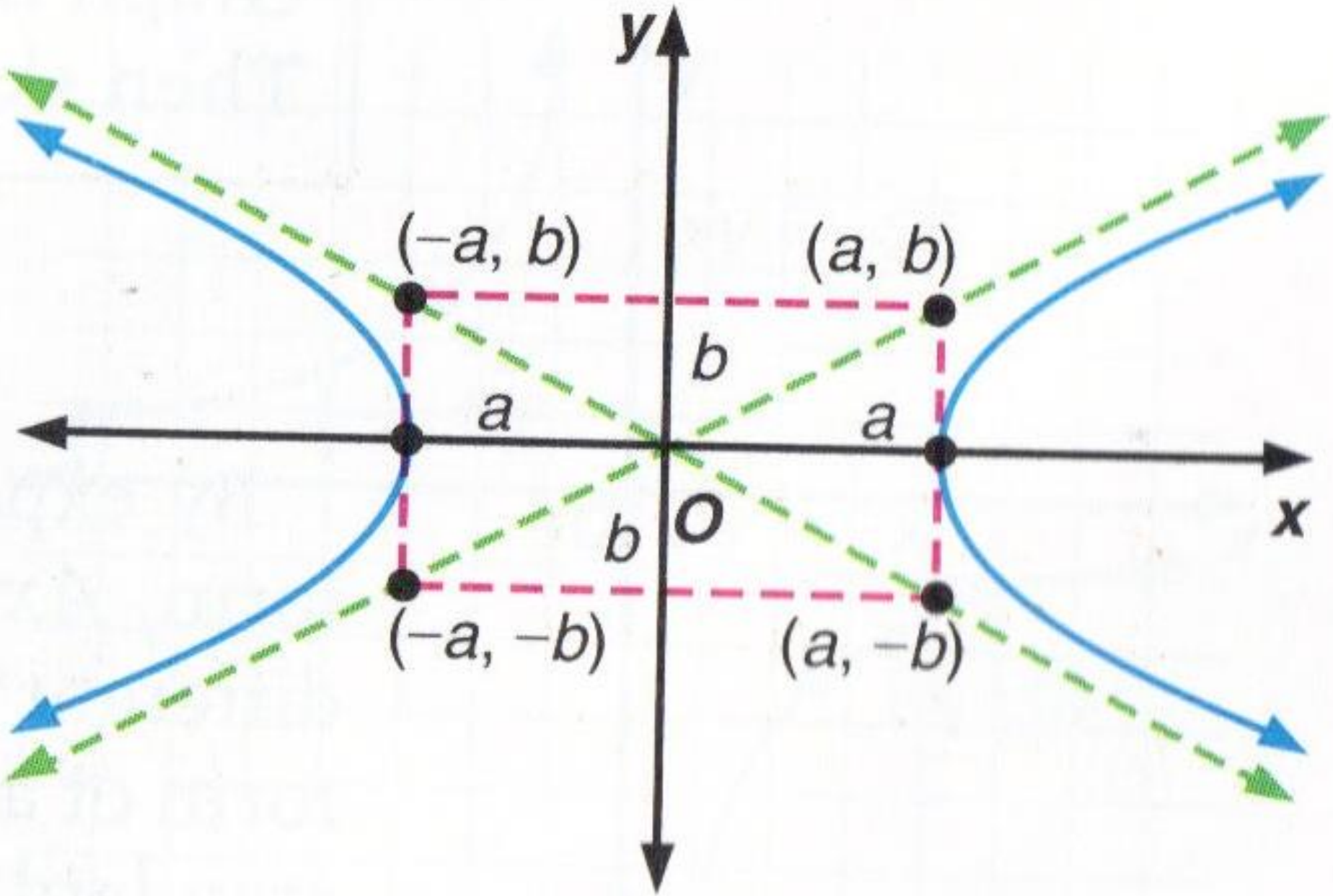
Example 1

- Sketch the graph of what you know.
- Find the center...find the midpoint of F_1F_2
- Find c, a^2 and b^2
- Use the standard form when the transverse is ll to the x-axis.

Graphing a hyperbola

- 1st graph the asymptotes

…these contain the diagonals of the rectangle defined by the transverese and conjugate axes.


y
(−a, b)
(a, b)
b
a
a
O
x
b
(−a, −b)
(a, −b)

- The x-coordinates of the vertices correspond to the x-coordinates of the endpoints of the transverse axis.
- Likewise, the y-coordinates correspond to the y-coordinates of the endpoints of the conjugate axis.
- The endpoints of the transverse and conjugate axes are midpoints of the sides of the rectangle.

Slopes of Asymptotes

- If the transverse axis lies along the x-axis the slopes are b/a and –b/a and both x intercepts are 0.
- If the transverse axis lies along the y-axis the slopes are a/b and –a/b and both x intercepts are 0.

Equations of the Asymptotes

$$y-k=\pm\frac{b}{a}(x-h)$$ When the transverse is // to the x-axis

$$y-k=\pm\frac{a}{b}(x-h)$$ When the transverse is // to the y-axis

Example 2

- Find the coordinates of the center, foci, and vertices and the equations of the asymptotes of the graph of the following and then sketch the graph.

$$\frac{(x-5)^2}{25} - \frac{(y+1)^2}{9} = 1$$

General Form

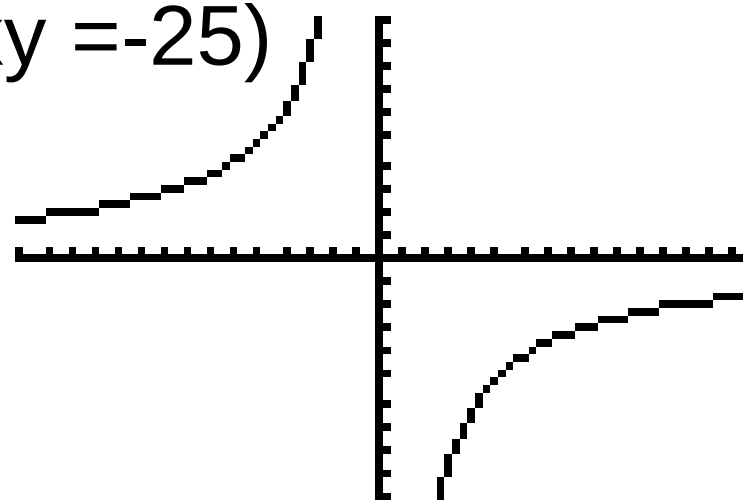
- $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where A and C are not zero and A and C have different signs.

Example 3

- Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph below and sketch.
- $4x^2 - y^2 + 24x + 4y + 28 = 0$

Equilateral Hyperbola

- when a=b in the standard form of a hyperbola
- The asymptotes are ⊥
- Special case is when these asymptotes are the x & y axes. (ie, xy =-25)



Example 4

- Graph xy = 36