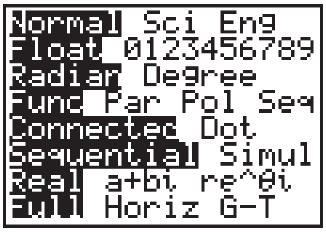
1 Introduction to the Graphing Calculator

This section introduces you to some commonly-used keys and menus of the calculator.

Setting Preferences

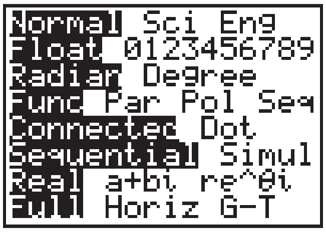
MODE The MODE key allows you to select your preferences in many aspects of calculation and graphing. Many of these settings are rarely changed in common usage. This screen shows the default mode settings.



← type of numeric notation
← number of decimal places in results
← unit of angle measure used
← type of graph (function, parametric, polar, sequence)
← whether to connect graphed points
← real, rectangular complex, or polar complex number system
← graph occupies full screen, top of screen with HOME screen below, or left side of screen with TABLE on right

To change the preferences, use the arrow keys to highlight your choice and press ENTER .

FORMAT The **FORMAT** menu is the second function of ZOOM and sets preferences for the appearance of your graphing screen. The default screen is shown below.



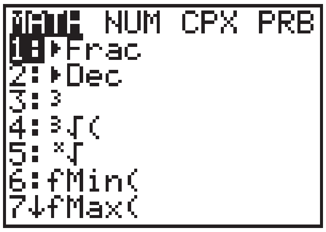
← rectangular or polar coordinate system
← whether to display the cursor coordinates on screen
← whether to show a grid pattern on screen
← whether to show the axes
← whether to label the axes
← whether to show the equation being graphed

You can change your preferences in the **FORMAT** menu in the same way you change MODE settings.

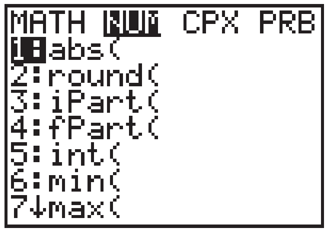
Using Menus

Many keys on the calculator access menus from which you can select a function, command, or setting. Some keys access multiple menus. You can use the right and left arrow keys to scroll through the different menu names located at the top of the screen. As each menu name is highlighted, the choices on the screen change. The screens on the next page show various menus accessed by using MATH .

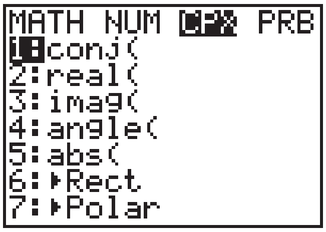
Math menu



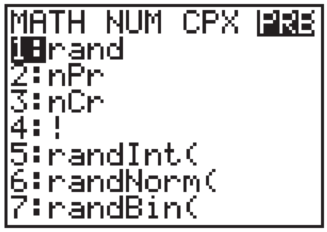
Number menu



Complex Number menu



Probability menu



To select a choice in a menu, either use the arrow keys to highlight your choice and press ENTER or simply press the number or letter of your selection. Notice that entry 7 in the first screen has a down arrow instead of a colon after the 7. This signifies there are more entries in the menu.

Alternate Function Keys

Whenever an alternate function is indicated in the keystrokes of this appendix, we will use brackets to show that the function is listed above a key.

Above most keys are one or two additional labels representing commands, menus, letters, lists, or operational symbols. These are accessed by using 2nd or ALPHA.

- 2nd accesses the commands on the left above the key. Note that these commands and 2nd are the same color.
- ALPHA accesses the commands on the right above each key. These commands and ALPHA are also the same color.
- Pressing 2nd ALPHA engages the **[A-LOCK]** or Alpha Lock command. This enables you to select consecutive ALPHA commands without pressing ALPHA before each command. This is especially useful when entering programs.

Each letter accessed by using ALPHA can be used to enter words or labels on the screen, but can also be used as a variable. A value can be stored to each variable.

Computation

A graphing calculator is also a scientific calculator. That is, it follows the order of operations when evaluating entries. Unlike some scientific calculators, the graphing calculator displays every entry in the expression.

Before pressing ENTER to evaluate the expression, you can use the arrow keys to scroll through the expression to make corrections. Corrections can be made in three ways.

- Use DEL to delete any unwanted entries.
- Use 2nd **[INS]** to insert omitted entries.
- "Type" over an incorrect entry. This overprints any entries and does not shift the entries to the right as a word processor does.

If you have an expression that you wish to evaluate repeatedly with a change in one part of the expression, you can press 2nd **[ENTRY]** after you have pressed ENTER and the expression will reappear. You can edit it for your next computation. The **ENTRY** command always repeats the last entered expression. You cannot scroll back through previous expressions you have evaluated.

Example

1 **Evaluate** $\dfrac{\sqrt{3^2 - 4(6) + [5 - (-12)]^3}}{6}$.

Press: 2nd [√‾] 3 x^2 — 4 (6) + (5 — (–) 12) ^ 3) ÷ 6 ENTER

The minus key and the negative key are different keys.

Note that the square root function automatically includes a left parenthesis. You must enter the right parenthesis to indicate the end of the expression under the radical sign. If you have the decimal in the **Float** mode, as many as 10 digits may appear in the answer.

```
√(3²-4(6)+(5--12
)^3)/6
      11.66428547
```

2 **Evaluate each expression if $a = 4$, $b = -5$, $c = 2$, $d = \frac{2}{3}$, and $e = -1.5$.**

a. $abc - 3de^4$

b. $\dfrac{e + 4a}{c^2 + 8b}$

For a series of expressions that use the same values for the variables, it is often helpful to store the value for each variable into the calculator. You can combine several commands in one line by using the colon after each command. The following commands save the values for variables a, b, c, d and e.

Press: 4 STO▸ ALPHA **[A]** ALPHA **[:]** (–) 5 STO▸ ALPHA **[B]** ALPHA **[:]** 2 STO▸ ALPHA **[C]** ALPHA **[:]** 2 ÷ 3 STO▸ ALPHA **[D]** ALPHA **[:]** (–) 1.5 STO▸ ALPHA **[E]** ENTER

a. Method 1: Using stored values

ALPHA **[A]** ALPHA **[B]** ALPHA **[C]** — 3 ALPHA **[D]** ALPHA **[E]** ^ 4 ENTER

Method 2: Entering computations

4 × (–) 5 × 2 — 3 × 2 ÷ 3 × ((–) 1.5) ^ 4 ENTER

```
ABC-3DE^4
          -50.125
4*-5*2-3*2/3*(-1
.5)^4
          -50.125
```

b. Method 1: Using stored values

(ALPHA **[E]** + 4 ALPHA **[A]**) ÷ (ALPHA **[C]** x^2 + 8 ALPHA **[B]**) ENTER

Method 2: Entering computations

((–) 1.5 + 4 × 4) ÷ (2 x^2 + 8 × (–) 5) ENTER

```
(E+4A)/(C²+8B)
     -.4027777778
(-1.5+4*4)/(2²+8
*-5)
     -.4027777778
█
```

2 Graphing Functions

Most functions can be graphed by using the [Y=] key. The viewing window most often used for non-trigonometric functions is the standard viewing window [−10, 10] scl:1 by [−10, 10] scl:1, which can be accessed by selecting **6:ZStandard** on the **ZOOM** menu. Then the window can be adjusted so that a **complete graph** can be viewed. A complete graph is one that shows the basic characteristics of the parent graph.

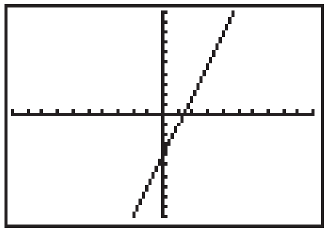
Example 1 **Linear Functions** A complete linear graph shows the x- and y-intercepts.

a. Graph $y = 3x - 4$ in the standard viewing window.

Press: [Y=] 3 [X,T,θ,n] [—] 4 [ZOOM] 6

If your calculator is already set for the standard viewing window, press [GRAPH] instead of [ZOOM] 6.

Both the x- and y-intercepts of the linear graph are viewable in this window, so the graph is complete.



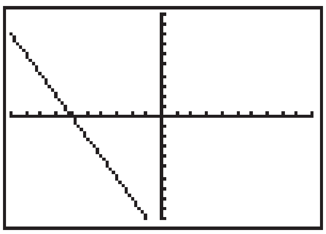
[−10, 10] scl:1 by [−10, 10] scl:1

b. Graph $y = -2(x + 5) - 2$.

Press: [Y=] [(−)] 2 [(] [X,T,θ,n] [+] 5 [)] [—] 2 [GRAPH]

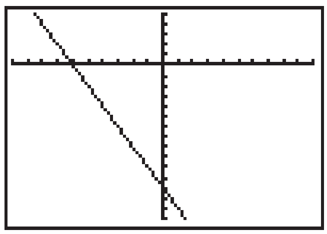
When this equation is graphed in the standard viewing window (Figure 1), a complete graph is not visible. The graph indicates that the y-intercept is less than −10. You can experiment with the **Ymin** setting or you can rewrite the equation in $y = mx + b$ form, which would be $y = -2x - 12$. The y-intercept is −12, so **Ymin** should be less than −12. Remember that **Xmax** and **Ymax** can be less than 10 so that your screen is less compressed. Use the **WINDOW** menu to change the parameters, or settings, and press [GRAPH] to view the result. There are many windows that will enable you to view the complete graph (Figure 2).

Figure 1



[−10, 10] scl:1 by [−10, 10] scl:1

Figure 2



[−10, 10] scl:1 by [−15, 5] scl:1

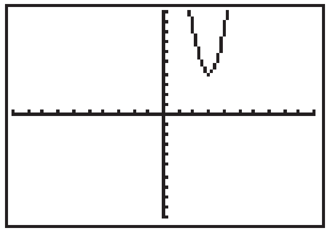
Example 2 **Quadratic Functions** When graphing quadratic functions, a complete graph includes the vertex of the parabola and enough of the graph to determine if it opens upward or downward.

(continued on the next page)

Graph $y = 4(x - 3)^2 + 4$.

Press: Y= 4 (X,T,θ,*n* — 3) x^2 + 4 GRAPH

While the standard viewing window shows a complete graph, you may want to change the viewing window to see more of the graph.



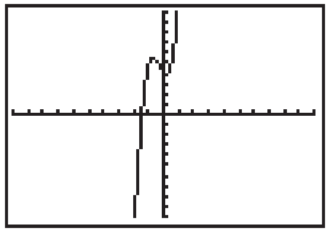
[−10, 10] scl:1 by [−10, 10] scl:1

Example 3 **Polynomial Functions** The graphs of other polynomial functions are complete when their maxima, minima, and x-intercepts are visible.

a. Graph $y = 5x^3 + 4x^2 - 2x + 4$.

Press: Y= 5 X,T,θ,*n* ^ 3 + 4 X,T,θ,*n* x^2 — 2 X,T,θ,*n* + 4 GRAPH

A complete graph is shown in the standard viewing window. You may want to redefine your window to observe the intercepts, maximum, and minimum points more closely.



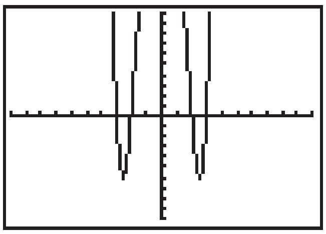
[−10, 10] scl:1 by [−10, 10] scl:1

b. Graph $y = x^4 - 13x^2 + 36$.

Press: Y= X,T,θ,*n* ^ 4 — 13 X,T,θ,*n* x^2 + 36 GRAPH

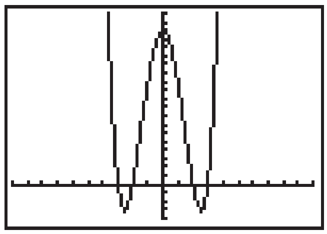
The standard viewing window (Figure 1) does not show a complete graph. It seems that only the y parameters need to be adjusted. Experiment to find a window that is suitable. Figure 2 shows a sample.

Figure 1



[−10, 10] scl:1 by [−10, 10] scl:1

Figure 2



[−10, 10] scl:1 by [−8, 40] scl:2

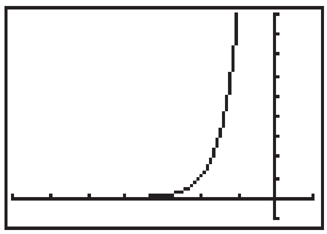
4 **Exponential Functions** A complete graph of an exponential function shows the curvature of the graph and the y-value that it approaches.

Graph $y = 9^{2+x}$.

Press: Y= 9 ^ (2 + X,T,θ,*n*) GRAPH

Note that you must use parentheses to group the terms that make up the exponent.

A complete graph seems to appear in the second quadrant of the standard viewing window. Vary the **WINDOW** settings to view the graph more closely.



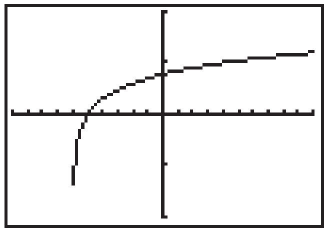
[−7, 1] scl:1 by [−1, 9] scl:1

Example 5 **Logarithmic Functions** A complete graph of a logarithmic function shows the curvature of the graph and the values, or locations, of the asymptotes that the curve approaches.

a. Graph $y = \log(x + 6)$.

Press: [Y=] [LOG] [X,T,θ,n] [+] 6 [)] [GRAPH]

An entire graph appears in the standard viewing window, but is very small. Redefine the **WINDOW** parameters for y, so that the graph is more visible.



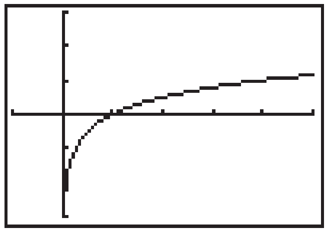
[−10, 10] scl:1 by [−2, 2] scl:1

b. Graph $y = \log_4 x$.

To graph a logarithmic function with a base other than 10, you must first change the function by using the change of base formula, $\log_a x = \frac{\log x}{\log a}$.

Press: [Y=] [LOG] [X,T,θ,n] [)] [÷] [LOG] 4 [)] [GRAPH]

An entire graph appears in the standard viewing window, but is very small. Redefine the **WINDOW** parameters, so that the graph is more visible.



[−1, 5] scl:1 by [−3, 3] scl:1

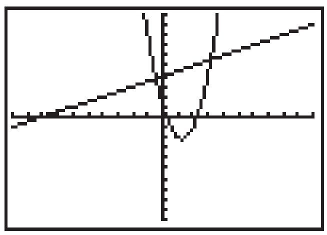
You can graph multiple functions on a single screen. Each function is denoted by **Y1=**, **Y2=**, **Y3=**, and so on, in the **Y=** menu. To graph more than one function, press [ENTER] at the end of each function you are entering and the cursor will move to the next function to be entered.

Example 6 **Systems of Equations**

Graph $y = 0.5x + 4$ and $y = 2x^2 - 5x + 1$.

Press: [Y=] 0.5 [X,T,θ,n] [+] 4 [ENTER] 2 [X,T,θ,n] [x^2] [—] 5 [X,T,θ,n] [+] 1 [GRAPH]

The standard viewing window shows that the line and parabola intersect in two points.



[−10, 10] scl:1 by [−10, 10] scl:1

GRAPHING CALCULATOR APPENDIX

3 Analyzing Functions

In addition to graphing a function, you can use other tools on a graphing calculator to analyze functions. One of those tools is a function table.

The function values are the dependent variable values.

Example 1 **How to Use a Table** You may complete a table manually or automatically. To create a table for one or more functions, you must first enter each function into the **Y=** list. Then set up and create the table.

a. Use a table to evaluate the function $y = 4x^2 - 2x + 7$ for {−9, −4, 0, 1, 5}.

In this case you only need to evaluate the function for selected values, so use the **TBLSET** menu to have the calculator ask for the values of the independent variable (domain) and find the function value (range) automatically.

Press: [Y=] 4 [X,T,θ,n] [x^2] [—] 2 [X,T,θ,n] [+]
7 [2nd] [TBLSET] [▼] [▼] [▶] [ENTER]
[2nd] [TABLE] [(–)] 9 [ENTER] [(–)]
4 [ENTER] 0 [ENTER] 1 [ENTER] 5 [ENTER]

X	Y1	
-9	349	
-4	79	
0	7	
1	9	
5	97	

X=

b. Use a table to evaluate the functions $y = 5x^2 - x + 1$ and $y = 6 - x^3$ for the integers from −3 to 3, inclusive.

When you want to evaluate a function for a range of values, have the calculator find both the values of the independent variable and the function values automatically. In **Table Setup**, enter the initial number of the domain as the **TblStart** value and the increment between the values of the independent variable as **ΔTbl**. Entering more than one function in the **Y=** list allows you to evaluate all of the functions in one table.

Press: [Y=] 5 [X,T,θ,n] [x^2] [—] [X,T,θ,n] [+]
1 [ENTER] 6 [—] [X,T,θ,n] [^] 3 [2nd]
[TBLSET] [(–)] 3 [ENTER] 1 [ENTER] [ENTER]
[2nd] [TABLE]

X	Y1	Y2
-3	49	33
-2	23	14
-1	7	7
0	1	6
1	5	5
2	19	-2
3	43	-21

X= -3

Once you create a table, you can scroll through the values using the arrow keys.

[ZOOM] allows you to quickly adjust the viewing window of a graph in different ways. The effect of each choice on the **ZOOM** menu is shown on the next page.

1: ZBox	Allows you to draw a box to define the viewing window
2: Zoom In	Magnifies the graph around the cursor
3: Zoom Out	Views more of a graph around the cursor
4: ZDecimal	Sets Δ**X** and Δ**Y** to 0.1
5: ZSquare	Sets equal-sized pixels on the *x*-and *y*-axes
6: ZStandard	Sets the standard viewing window, [−10, 10] scl:1 by [−10, 10] scl:1
7: ZTrig	Sets the built-in trig window, $\left[-\frac{47}{24}\pi, \frac{47}{24}\pi\right]$ scl: $\frac{\pi}{2}$ by [−4, 4] scl:1 for radians or [−352.5. 352.5] scl:90 by [−4, 4] scl: 1 for degrees
8: ZInteger	Sets integer values on both the *x*-and *y*-axes
9: ZoomStat	Sets values for displaying all of the data in the current stat lists
0: ZoomFit	Fits Ymin and Ymax to show all function values for Xmin to Xmax

Example 2 **Using [ZOOM] to Graph in the Standard and Square Windows**

Graph the circle with equation $x^2 + y^2 = 16$ in the standard viewing window. Then use ZSquare to view the graph in a square screen.

First solve the equation for y in order to enter it into the **Y=** list.

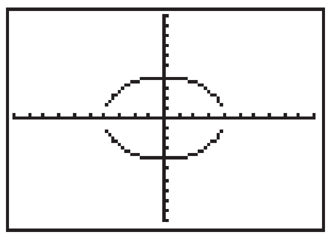
$x^2 + y^2 = 16 \rightarrow y = \pm\sqrt{16 - x^2}$

The two pieces of the graph can be entered at one time using {−1, 1}. This expression tells the calculator to graph −1 and 1 times the function.

Press: [Y=] [2nd] [{] [(–)] 1 [,] 1 [2nd] [}] [2nd] [√] 16 [—] [X,T,θ,n] [x²] [ )] [ZOOM] 6

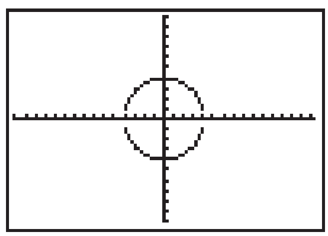
The circle is distorted when viewed in the standard viewing window.

Press: [ZOOM] 5



[−10, 10] scl:1 by [−10, 10] scl:1

Using **ZSquare** makes the circle appear as a circle.



[−15.16, 15.16] scl:1 by [−10, 10] scl:1

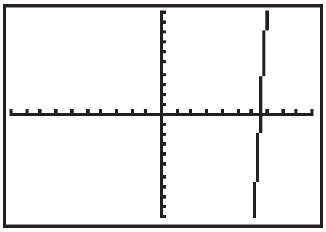
Example 3 **Using [ZOOM] to Zoom In and Out** **Graph $y = 0.5x^3 - 3x^2 - 12$ in the standard viewing. Zoom out to view a complete graph. Then zoom in to approximate the y-intercept of the graph to the nearest whole number.**

Press: [Y=] 0.5 [X,T,θ,n] [^] 3 [—] 3 [X,T,θ,n] [x^2] [—] 12 [ZOOM] 6

The complete graph is not shown in the standard viewing window. (Figure 1) When you zoom out or in, the calculator allows you to choose the point around which it will zoom. Zooming out around the origin once allows a complete graph to be shown. (Figure 2)

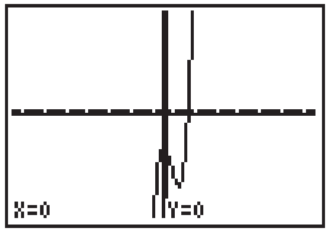
Press: [ZOOM] 3 [ENTER]

Figure 1



[−10, 10] scl:1 by [−10, 10] scl:1

Figure 2

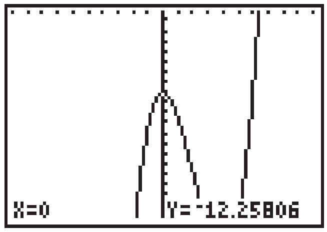


[−40, 40] scl:1 by [−40, 40] scl:1

Now zoom in to approximate the y-intercept. Choose a point close to the intercept by using the arrow keys.

Press: [ZOOM] 2 [ENTER]

The y-intercept appears to be about −12. Zooming in again may allow you to make a closer approximation.



[−10, 10] scl:1 by [−24.19, −4.19] scl:1

The [TRACE] feature allows you to move the cursor along a graph and display the coordinates of the points on the graph.

Example 4 **Using [TRACE]** **Graph $y = 4x + 2$ and $y = -3x^2 - x + 5$. Use the TRACE feature to approximate the coordinates of the intersection of the graphs in the first quadrant. Then evaluate $y = -3x^2 - x + 5$ for $x = 1.7$.**

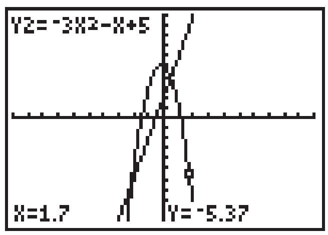
Make sure that **CoordOn** *is highlighted in the* **FORMAT** *menu to display the cursor coordinates as you trace.*

Press: [Y=] 4 [X,T,θ,n] [+] 2 [ENTER] [(–)] 3 [X,T,θ,n] [x^2] [—] [X,T,θ,n] [+] 5 [TRACE]

Move the cursor along the graphs using [◀] and [▶].

Pressing [2nd] [◀] or [2nd] [▶] moves the cursor more quickly. If your cursor moves off of the screen, the calculator will automatically update the viewing window so that the cursor is visible. Use [▼] and [▲] to move from one function to the other. The intersection is at about (0.4, 4).

To evaluate a function for a value and move to that point, place the cursor on the function graph. Then enter the value and press [ENTER]. When $x = 1.7$, $y = -5.37$ for $y = -3x^2 - x + 5$.



[−10, 10] scl:1 by [−10, 10] scl:1

Using [TRACE] to locate the intersection points of the graphs of two functions gives you an approximation of the coordinates. For more accurate coordinates, you can use the **intersect** option on the **CALC** menu.

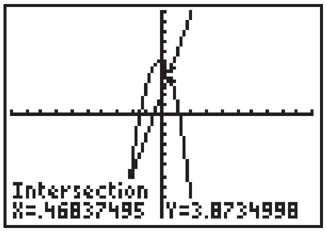
Example 5 **Finding Intersection Points** **Use 5:intersection on the CALC menu to find the coordinates of the intersection of the graphs of $y = 4x + 2$ and $y = -3x^2 - x + 5$.**

If you do not have the functions graphed, enter the functions into the **Y=** list and press [GRAPH]. Then find the coordinates of the intersection.

Press: [2nd] **[CALC]** 5

The intersection of the graphs must appear on the screen to find the coordinates when using **intersect**.

Place the cursor on one graph and press [ENTER]. Then move the cursor to the other graph and press [ENTER]. To guess at the intersection, move the cursor to a point close to the intersection or enter an x-value and press [ENTER]. If there is more than one intersection point, the calculator will find the one closest to your guess. The cursor will move to the intersection point and the coordinates will be displayed.



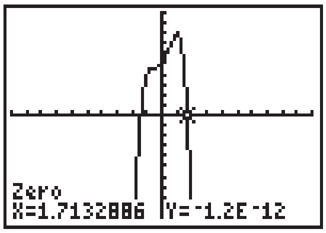
[−10, 10] scl:1 by [−10, 10] scl:1

The **CALC** menu also allows you to find the zeros of a function.

Example 6 **Finding Zeros** **Find the zeros of $f(x) = -2x^4 + 3x^2 + 2x + 5$.**

Press: [Y=] [(–)] 2 [X,T,θ,n] [^] 4 [+] 3 [X,T,θ,n] [x^2] [+] 2 [X,T,θ,n] [+] 5 [2nd] **[CALC]** 2

The calculator can find one zero at a time. Use the arrow keys or enter a value to choose the left bound for the interval in which the calculator will search for the zero and press [ENTER]. Choose the right bound and press [ENTER]. Select a point near the zero using the arrow keys or by entering a value and press [ENTER]. Repeat with another interval to find the other zero. The zeros of this function are about −1.42 and 1.71.



[−10, 10] scl:1 by [−10, 10] scl:1

Real-world application problems often require you to find the relative minimum or maximum of a function. You can use **3:minimum** or **4:maximum** features on the **CALC** menu of a graphing calculator to solve these problems.

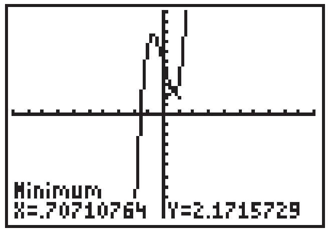
Example 7 Finding Maxima and Minima **Determine the relative minimum and the relative maximum for the graph of $f(x) = 4x^3 - 6x + 5$.**

First graph the function.

Press: [Y=] 4 [X,T,θ,n] [^] 3 [—] 6 [X,T,θ,n] [+] 5 [ZOOM] 6

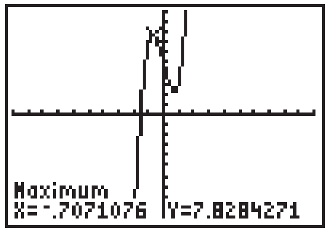
To find the relative minimum press [2nd] **[CALC]** 3.

Similar to finding a zero, choose the left and right bound of the interval and guess the minimum or maximum. The point at about (0.71, 2.17) is a relative minimum.



[−10, 10] scl:1 by [−10, 10] scl:1

Use a similar method to find the relative maximum, by pressing [2nd] **[CALC]** 4. The point at about (−0.71, 7.83) is a relative maximum.



[−10, 10] scl:1 by [−10, 10] scl:1

4 Graphing Inequalities

Most linear and nonlinear inequalities can be graphed using the [Y=] key and selecting the appropriate graph style in the **Y=** editor. To select the appropriate graph style, select the graph style icon in the first column of the **Y=** editor and press [ENTER] repeatedly to rotate through the graph styles.

- To shade the area above a graph, select the Above style icon, ◥.
- To shade the area below a graph select the Below style icon, ◣.

Before graphing an inequality, clear any functions in the **Y=** list by pressing [Y=] and then using the arrow keys and the [CLEAR] key to select and clear all functions. If you do not wish to clear a function, you can turn that particular graph off by using the arrow keys to position the cursor over that function's = sign and then pressing [ENTER] to change the selection status.

Example 1 Linear Inequalities

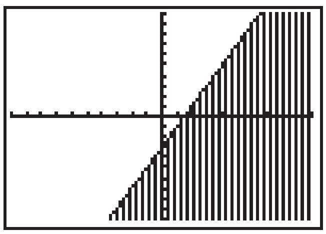
a. Graph $y \leq 2x - 3$ in the standard viewing window.

First enter the boundary equation $y = 2x - 3$ into the **Y=** list.

Press: [Y=] 2 [X,T,θ,n] [—] 3

Next, press the [◀] key until the icon before = flashes. Press [ENTER] until the icon changes to the Below style icon, ◣, for "$y \leq$".

Finally, if your calculator is not already set for the standard viewing window, press [ZOOM] 6. Otherwise, press [GRAPH].



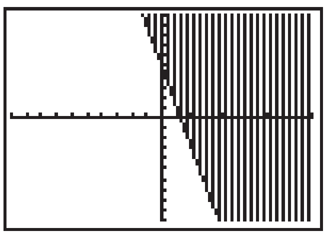
[−10, 10] scl:1 by [−10, 10] scl:1

b. Graph $y \geq -4x + 5$ in the standard viewing window.

Press: [Y=] [(–)] 4 [X,T,θ,n] [+] 5

Next, press the [◀] key until the icon before = flashes. Then press [ENTER] until the icon changes to the Above style icon, ◥, since the inequality asks for "$y \geq$".

Finally, press [GRAPH].



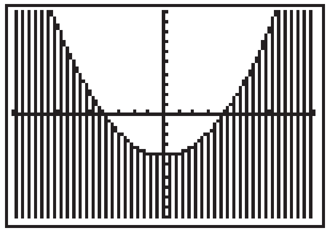
[−10, 10] scl:1 by [−10, 10] scl:1

Example 2 **Nonlinear Inequalities** The procedure for graphing nonlinear inequalities is the same as that of graphing linear inequalities.

a. Graph $y \leq 0.25x^2 - 4$ in the standard viewing window.

Press: [Y=] 0.25 [X,T,θ,n] [x^2] [—] 4

Next, select the Below style icon, ◣, since the inequality asks for "$y \leq$", and then press [GRAPH].

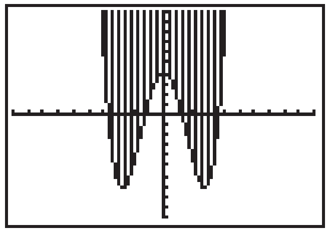


[−10, 10] scl:1 by [−10, 10] scl:1

b. Graph $y \geq 0.2x^4 - 3x^2 + 4$.

Press: [Y=] 0.2 [X,T,θ,n] [^] 4 [—] [X,T,θ,n] [^] 2 [+] 4

Next, select the Above style icon, ◥, since the inequality asks for "$y \geq$". Then press [GRAPH].

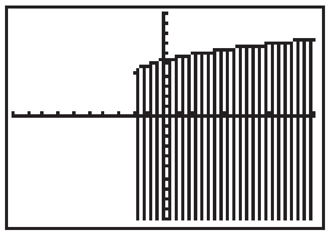


[−10, 10] scl:1 by [−10, 10] scl:1

c. Graph $y \leq \sqrt{x + 2} + 4$.

Press: [Y=] [2nd] [√‾] [X,T,θ,n] [+] 2 [)] [+] 4

Next, select the Below style icon, ◣, since the inequality asks for "$y \leq$". Then press [GRAPH].

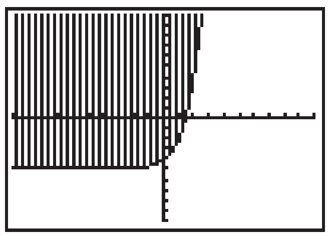


[−10, 10] scl:1 by [−10, 10] scl:1

d. Graph $y \geq 3^x - 5$.

Press: [Y=] 3 [^] [X,T,θ,n] [—] 5

Next, select the Above style icon, ◥, since the inequality asks for "$y \geq$". Then press [GRAPH].



[−10, 10] scl:1 by [−10, 10] scl:1

Graphing systems of inequalities on a graphing calculator is similar to graphing systems of equations.

Example 3 **Graph the system of inequalities.**

$y \geq 2x - 5$

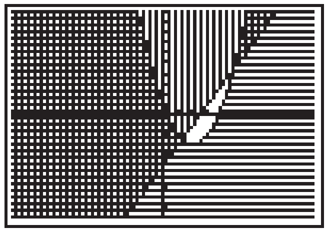
$y \leq x^2 - 4x + 1$

Method 1: Shading Options in **Y=**

Press: [Y=] 2 [X,T,θ,n] [—] 5 [ENTER] [X,T,θ,n] [x^2] [—] 4 [X,T,θ,n] [+] 1

Select the Above style icon, ◥, for $y \geq 2x - 5$ and the Below style icon, ◣, for $y \leq x^2 - 4x + 1$. Then press [GRAPH] .

Notice that the first inequality is indicated using vertical lines and the second inequality uses horizontal lines. The solution to the system is shown by the intersection of the shaded areas.



[−10, 10] scl:1 by [−10, 10] scl:1

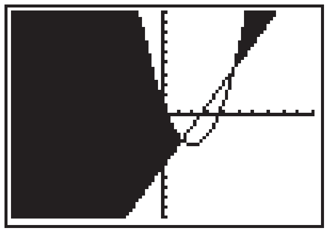
Method 2: Using the Shade Command

The **Shade(** *command can only be used with two inequalities which can be written with "$y \leq$" in one inequality and "$y \geq$" in the other.*

Some systems of inequalities can be graphed by using the **Shade(** command and entering a function for a lower boundary and a function for the upper boundary of the inequality. The calculator first graphs both functions and then shades above the first function entered and below the second function entered.

Before graphing an inequality using the **Shade(** command, clear any graphics from the viewing window by pressing [2nd] **[DRAW]** 1 [ENTER] . Also clear any equations in the **Y=** list. If not already there, return to the home screen by pressing [2nd] **[QUIT]**.

Press: [2nd] **[DRAW]** 7 2 [X,T,θ,n] [—] 5 [,] [X,T,θ,n] [x^2] [—] 4 [X,T,θ,n] [+] 1 [)] [ENTER]



[−10, 10] scl:1 by [−10, 10] scl:1

For more shading pattern options, see page 685.

5 Matrices

A graphing calculator can perform operations with matrices. It can also find determinants and inverses of matrices. The **MATRX** menus are accessed using [2nd] **[MATRX]**.

There are three menus in the **MATRX** menu.

- The **NAMES** menu lists the matrix locations available. There are ten matrix variables, [A] through [J].
- The **MATH** menu lists the matrix functions available.
- The **EDIT** menu allows you to define matrices.

A matrix with dimension 2 × 3 indicates a matrix with 2 rows and 3 columns. Depending on available memory, a matrix may have up to 99 rows or columns.

Example 1 Entering a Matrix

Enter matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$.

To enter a matrix into your calculator, choose the **EDIT** menu and select the matrix name. Then enter the dimensions and elements of the matrix.

Press: [2nd] [MATRX] [◀] [ENTER] 2 [ENTER] 2 [ENTER] 1 [ENTER] 3 [ENTER] 2 [ENTER] (-) 2

Press [2nd] **[QUIT]** to return to the **HOME** screen. Then press [2nd] **[MATRX]** [ENTER] [ENTER] to display the matrix.

```
[A]
          [[1  3 ]
           [2  -2]]
```

You can find the determinant and inverse of a matrix very quickly with a graphing calculator.

Example 2 Determinant and Inverse of a Matrix

a. Find the determinant of matrix *A*.

Press: [2nd] **[MATRX]** [▶] 1 [2nd] **[MATRX]** 1 [ENTER]

The determinant of matrix A is -8.

```
det([A]
                -8
```

b. Find the inverse of matrix *A*.

Press: [2nd] **[MATRX]** 1 [x^{-1}] [ENTER]

$$A^{-1} = \begin{bmatrix} 0.25 & 0.375 \\ 0.25 & -0.125 \end{bmatrix}$$

```
[A]-1
     [[.25  .375 ]
      [.25  -.125]]
```


CONTENTS

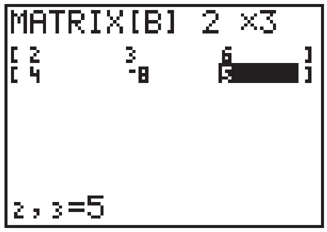
Example 3 Operations with Matrices

Enter matrix $B = \begin{bmatrix} 2 & 3 & 6 \\ 4 & -8 & 5 \end{bmatrix}$. Then perform each operation.

a. $\frac{1}{2}B$

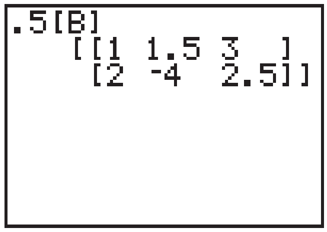
First, enter matrix *B*.

Press: [2nd] [MATRX] [◀] 2 2 [ENTER]
3 [ENTER] 2 [ENTER] 3 [ENTER] 6 [ENTER]
4 [ENTER] [(–)] 8 [ENTER] 5 [2nd] [QUIT]



Then find $\frac{1}{2}B$.

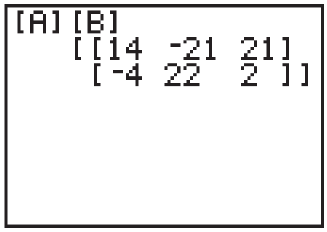
Press: .5 [2nd] [MATRX] 2 [ENTER]



$$\frac{1}{2}B = \begin{bmatrix} 1 & 1.5 & 3 \\ 2 & -4 & 2.5 \end{bmatrix}$$

b. ***AB***

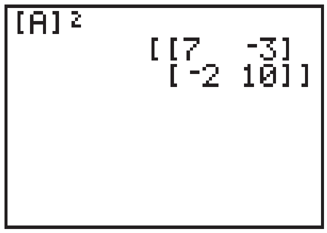
Press: [2nd] [MATRX] 1 [2nd] [MATRX] 2 [ENTER]



$$AB = \begin{bmatrix} 14 & -21 & 21 \\ -4 & 22 & 2 \end{bmatrix}$$

c. A^2

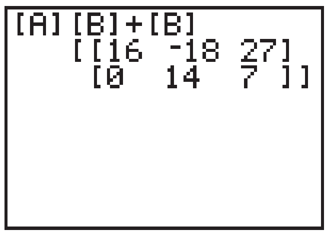
Press: [2nd] [MATRX] 1 [x^2] [ENTER]



$$A^2 = \begin{bmatrix} 7 & -3 \\ -2 & 10 \end{bmatrix}$$

d. ***AB* + *B***

Press: [2nd] [MATRX] 1 [2nd] [MATRX] 2 [+]
[2nd] [MATRX] 2 [ENTER]



$$AB + B = \begin{bmatrix} 16 & -18 & 27 \\ 0 & 14 & 7 \end{bmatrix}$$

6 Graphing Trigonometric Functions

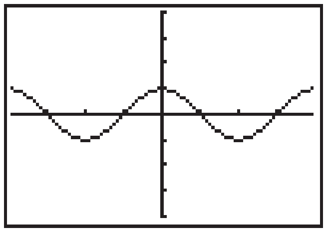
Trigonometric functions and the inverses of trigonometric functions can be graphed using [Y=]. The functions and their inverses can be graphed in degrees or radians. You must set the calculator in **Radian** or **Degree** mode. The standard viewing window for trigonometric functions can be set by pressing [ZOOM] **7:Trig**, which automatically adjusts the *x*- and *y*-axes scales for degrees or radians.

Example 1

a. Using Degrees **Graph $y = \cos x$.**

First, set the calculator in degree mode by pressing [MODE] [▼] [▼] [▶] [ENTER].

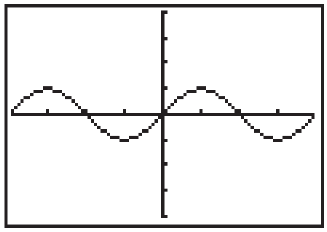
Now enter and graph the function. Press [Y=] [COS] [X,T,θ,n] [(–)] [ZOOM] 7.



[−352.5, 352.5] scl:90 by [−4, 4] scl:1

b. Using Radians **Graph $y = \sin x$.**

Change to radian mode by pressing [MODE] [▼] [▼] [ENTER]. Press [Y=] [CLEAR] to delete the function entered in part **a**. Then press [SIN] [X,T,θ,n] [(–)] to enter the new function. Next, press [ZOOM] 7 to set the viewing window to accommodate radian mode and graph the function.

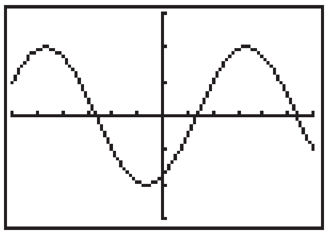


$[-2\pi, 2\pi]$ scl:$\frac{\pi}{2}$ by [−4, 4] scl:1

c. Amplitude, Period, and Phase Shift **Graph $y = 2 \sin\left(\frac{1}{2}x - 60°\right)$ using the viewing window [−540, 540] scl:90 by [−3, 3] scl:1. Then state the amplitude, period, and phase shift.**

Make sure the calculator is in degree mode.

Press: [Y=] 2 [SIN] [X,T,θ,n] [÷] 2 [—] 60 [)] [GRAPH]



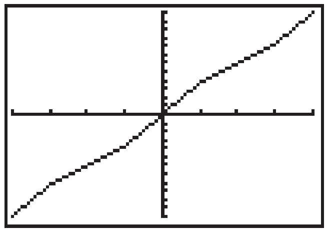
[−540, 540] scl:90 by [−3, 3] scl:1

The amplitude of the function is $\frac{|-2-2|}{2}$ or 2. The period is $\frac{360}{\frac{1}{2}}$ or 720°, and the phase shift is $\frac{-60}{\frac{1}{2}}$ or −120°.

d. Graph $y = 3x + \sin x$.

Use radian mode and the viewing window $[-2\pi, 2\pi]$ scl: $\frac{\pi}{2}$ by $[-6\pi, 6\pi]$ scl: $\frac{\pi}{2}$. You can enter the viewin\g window using [2nd] [π]. The calculator will determine a decimal approximation.

Press: [Y=] 3 [X,T,θ,n] [+] [SIN] [X,T,θ,n] [GRAPH]



$[-2\pi, 2\pi]$ scl:$\frac{\pi}{2}$ by $[-6\pi, 6\pi]$ scl:$\frac{\pi}{2}$

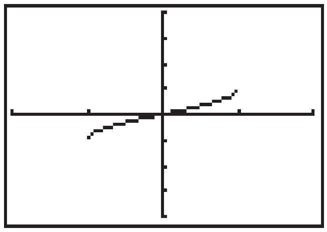
Because a graphing calculator only graphs functions, the graph of each inverse is limited to the domain for which the inverse of the function is defined.

Example 2 Inverses of Trigonometric Functions

a. Graph $y = \text{Arcsin } x$.

Choose an appropriate viewing window for degree or radian mode.

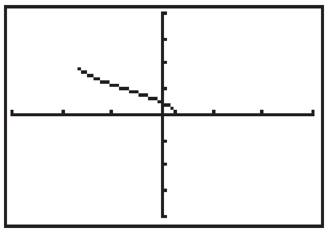
Press: [Y=] [2nd] [$\sin^{-1}$] [X,T,θ,n] [)] [GRAPH]



$[-2, 2]$ scl:1 by $[-360, 360]$ scl:90 or $[-2, 2]$ scl:1 by $[-2\pi, 2\pi]$ scl:$\frac{\pi}{2}$

b. Graph $y = \text{Cos}^{-1}\left(\frac{\sqrt{2}}{2} + x\right)$.

Press: [Y=] [2nd] [$\cos^{-1}$] [2nd] [$\sqrt{\ }$] 2 [)] [÷] 2 [+] [X,T,θ,n] [)] [GRAPH]



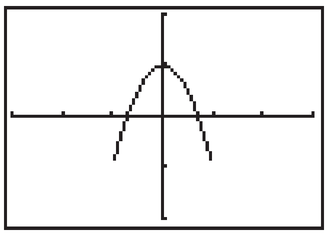
$[-3, 3]$ scl:1 by $[-360, 360]$ scl:90 or $[-3, 3]$ scl:1 by $[-2\pi, 2\pi]$ scl:$\frac{\pi}{2}$

c. Graph $y = \cos(2\,\text{Sin}^{-1} x)$.

Use the viewing window $[-3, 3]$ scl:1 by $[-2, 2]$ scl:1.

Press [Y=] [COS] 2 [2nd] [$\sin^{-1}$] [X,T,θ,n] [)] [)] [GRAPH]

The result is the same whether using degree or radian mode.



$[-3, 3]$ scl:1 by $[-2, 2]$ scl:1

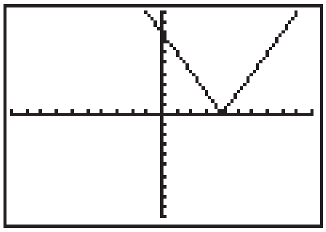
7 Graphing Special Functions

Most special functions can be graphed using the [Y=] key. The absolute value function **abs(** and the greatest integer function **int(** can be found in the **MATH NUM** menu.

Example 1 Absolute Value

Graph $y = 2|x - 4|$.

Press [Y=] 2 [MATH] [▶] 1 [X,T,θ,n] [—] 4 [)] [ZOOM] 6 to graph the function in the standard viewing window.



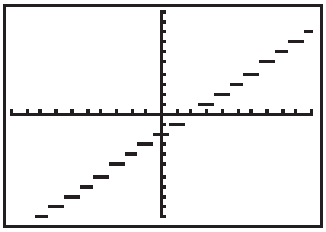
[−10, 10] scl:1 by [−10, 10] scl:1

2 Greatest Integer Function

Graph $y = [\![x - 1.5]\!]$.

First, make sure the calculator is set for dot plotting rather than the connected plotting used in most other functions. Press [MODE], highlight **Dot**, and press [ENTER].

Then, enter the function. Press [Y=] [MATH] [▶] 5 [X,T,θ,n] [—] 1.5 [)] [ZOOM] 6. If your calculator is already set for the standard viewing window, press [GRAPH] instead of [ZOOM] 6.



[−10, 10] scl:1 by [−10, 10] scl:1

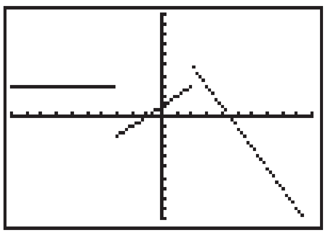
The **TEST** menu allows you to graph other piecewise functions. Enter the pieces of the function as a sum of the products of each piece of the function and its domain. For example, $y = \begin{cases} 5 \text{ if } x < 2 \\ 4x \text{ if } x > 2 \end{cases}$ is entered as **(5)(X<2) + (4X)(X>2)** in the **Y=** menu.

Example 3 Piecewise Function

Graph $y = \begin{cases} 3 \text{ if } x \leq -3 \\ 1 + x \text{ if } -3 < x \leq 2. \\ 9 - 2x \text{ if } x > 2 \end{cases}$

Place the calculator in **Dot** mode. Then enter the function in the **Y=** list using the **TEST** menu options.

Press: [Y=] [(] 3 [)] [(] [X,T,θ,n] [2nd] **[TEST]** 6 [(−)] 3 [)] [+] [(] 1 [+] [X,T,θ,n] [)] [(] [(−)] 3 [2nd] **[TEST]** 5 [X,T,θ,n] [)] [(] [X,T,θ,n] [2nd] **[TEST]** 6 2 [)] [+] [(] 9 [—] 2 [X,T,θ,n] [)] [(] [X,T,θ,n] [2nd] **[TEST]** 3 2 [)] [ZOOM] 6



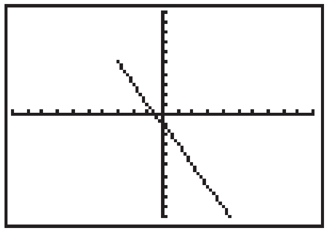
[−10, 10] scl:1 by [−10, 10] scl:1

8 Parametric and Polar Equations

Parametric and polar equations can be graphed using the Y= key by selecting the **Par** or **Pol** setting in the **MODE** menu.

Example 1 **Parametric Equations** **Graph the parametric equations $x = -3 + 2t$ and $y = 5 - 4t$.**

First, set the calculator to parametric mode by pressing MODE ▼ ▼ ▼ ▶ ENTER . Then press WINDOW 0 ENTER 10 ENTER 0.5 ENTER (–) 10 ENTER 10 ENTER 1 ENTER (–) 10 ENTER 10 ENTER 1 ENTER to set the viewing window.



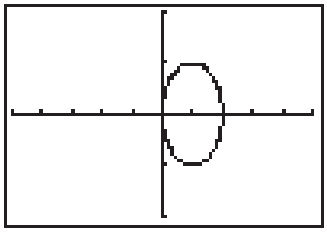
[−10, 10] tstep:0.5, [−10, 10] scl:1 by [−10, 10] scl:1

Enter the parametric equations.

Press: Y= (–) 3 + 2 X,T,θ,n ENTER 5 — 4 X,T,θ,n GRAPH

2 **Polar Equations** **Graph $r = 2 \cos \theta$.**

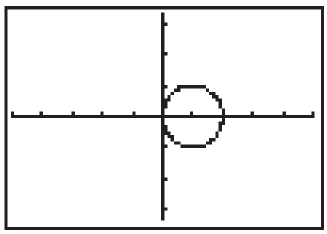
Set the calculator in polar mode. Press MODE ▼ ▼ ▼ ▶ ▶ ENTER , and make sure the calculator is in radian mode. Then set the viewing window by pressing WINDOW 0 ENTER 2nd [π] ENTER 0.05 ENTER (–) 5 ENTER 5 ENTER 1 ENTER (–) 2 ENTER 2 ENTER 1 ENTER .



[0, 2π] θstep:0.05, [−5, 5] scl:1 by [−2, 2] scl:1

Enter the equation. Press Y= 2 COS X,T,θ,n) GRAPH .

The graph appears to be an ellipse, but pressing ZOOM 5 to set a square viewing window shows that it is actually a circle.



[0, π] θstep:0.05, [−5, 5] scl:1 by [−3.29787234, 3.29787234] scl:1

GRAPHING CALCULATOR APPENDIX

9 Statistics and Statistical Graphs

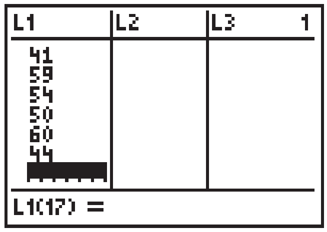
A graphing calculator allows you to enter a set of data and generate statistics and statistical graphs. Before you enter data values, make sure you clear the **Y=** list, **L1** and **L2**, and the graphics screen. Clear the **Y=** list by pressing [Y=] [CLEAR]. Use the [▼] key to select additional equations and clear them also. To clear **L1** and **L2**, press [STAT] 4 [2nd] **[L1]** [,] **[L2]** [ENTER]. If you need to clear the graphics screen, press [2nd] **[DRAW]** 1 [ENTER].

Example

1 Enter Data into Lists **Enter the following data into a graphing calculator.**

49 53 54 54 56 55 57 61 51 58 41 59 54 50 60 44

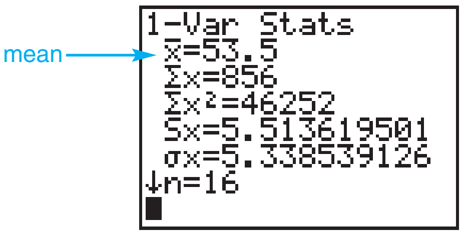
Press: [STAT] 1 53 [ENTER] 54 [ENTER] 54 [ENTER]
56 [ENTER] 55 [ENTER] 57 [ENTER] 61 [ENTER]
51 [ENTER] 58 [ENTER] 41 [ENTER] 59 [ENTER]
54 [ENTER] 50 [ENTER] 60 [ENTER] 44 [ENTER]

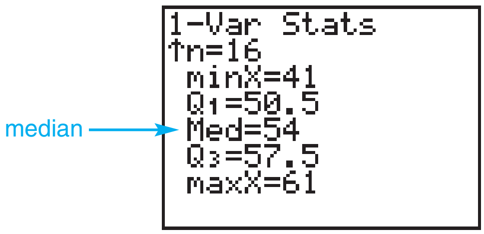


You can use the up and down arrow keys to scroll through the list.

2 Find Mean, Median, and Mode **Find the mean, median, and mode of the data in Example 1.**

Press [STAT] [▶] 1 [ENTER]. This function displays many statistics about the data. $\bar{X}$ denotes the mean. Scroll down to find the median.





The calculator does not have a function to determine the mode. You can find the mode by examining the data. First sort the data to write them in order from least to greatest.

Press: [STAT] 2 [2nd] **[L1]** [)] [ENTER]

Then scroll through the data by pressing [STAT] 1 and using the [▲] and [▼] keys. You will find that the mode is 54.

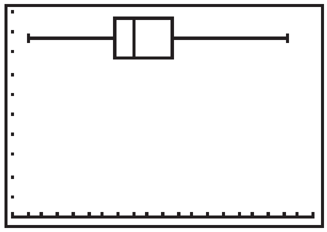
Examples

3 Box-and-Whisker Plots

a. Draw a box-and-whisker plot for the data.

30.2	29.0	26.2	25.8	23.8	43.0	19.8	19.4
26.0	46.6	26.8	22.8	35.4	25.2	12.2	31.4

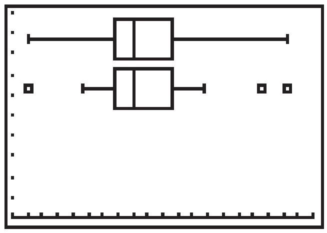
Set the viewing window. Next, set the plot type. Press 2nd [STAT PLOT] 1 ENTER ▼ ▶ ▶ ▶ ▶ to highlight and press ENTER. Make sure **L1** is entered in **Xlist:**. If not, move the cursor to highlight and press 2nd [L1] ENTER. Then, enter the data into **L1**. Press STAT 1 30.2 ENTER 29 ENTER … 31.4 ENTER GRAPH.



[10, 50] scl:2 by [0, 10] scl:1

b. Draw a box-and-whisker plot with outliers using the data.

Without clearing the lists or graphic screen, press 2nd [STAT PLOT] 2 ENTER ▼ ▶ ▶ ▶, to highlight, and press ENTER. Make sure **L1** is entered in **Xlist:**. Press GRAPH.



[10, 50] scl:2 by [0, 10] scl:1

c. Find the upper and lower quartiles, the median, and the outliers.

Press TRACE and use ◀ and ▶ to move the cursor along the graph. The values will be displayed. For this data, the upper quartile is 30.8, the lower quartile is 23.3, the median is 26.1, and the outliers are 43 and 46.6.

4 Histograms

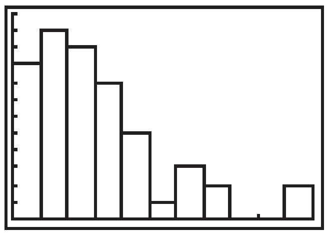
a. Use the data on the number of public libraries in each state and Washington, D.C., to make a histogram.

273	102	159	196	1030	235	244	30	27	428	366	49	141
772	427	554	372	188	322	273	187	491	659	361	243	346
110	283	78	238	455	92	1067	352	86	684	192	201	640
74	180	134	284	753	96	204	308	309	174	451	74	

Enter the data in **L1**. Press STAT 1 273 ENTER 102 ENTER … 74 ENTER.

Set the viewing window. Choose **Xmin**, **Xmax**, and **Xscl** to determine the number of bars in the histogram. For this data, the least value is 27 and the greatest is 1067. If **Xmin** = 0, **Xmax** = 1100, and **Xscl** = 100, the histogram will have 11 bars each representing an interval of 100.

Choose the type of graph. Press 2nd [STAT PLOT] 1 ENTER ▼ ▶ ▶ ENTER ▼ 2nd [L1] ENTER ▼ 1 ENTER. Then press GRAPH to draw the histogram.



[0, 1100] scl:100 by [0, 12] scl:1

Make sure you have cleared the **Y=** *list,* **L1** *and* **L2**, *and the graphic screen.*

b. Use the data in the table to draw a histogram and its frequency polygram.

Class Limits	Frequency
155–170	7
170–185	15
185–200	34
200–215	38
215–230	42
230–245	35
245–260	33
260–275	21
275–290	18
290–305	6
305–320	3

Enter the class marks as **L1**. Press [STAT] 1 162.5 [ENTER] 177.5 [ENTER] … 312.5 [ENTER] .

Move the cursor to **L2**. Enter the frequencies.

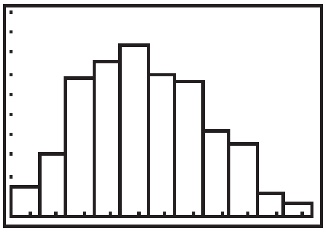
Press 7 [ENTER] 15 [ENTER] … 3 [ENTER] [2nd] **[QUIT]**.

Set the viewing window. Use the minimum and maximum of the class limits for **Xmin** and **Xmax**. Use the size of the intervals for **Xscl**. Choose the y-axis values to show the complete histogram.

Set the plot type. Press [2nd] **[STAT PLOT]** 1 [ENTER] [▼] [▶] [▶] [ENTER] [▼] [2nd] **[L1]** [ENTER] [▼] [2nd] **[L2]** [ENTER] . Then press [GRAPH] .(Figure 1)

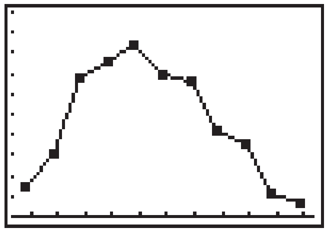
Without clearing the lists or graphic screen, press [2nd] **[STAT PLOT]** 1 [ENTER] [▼] [▶] to highlight the line plot icon, and press [ENTER] . Make sure **L1** is entered in **Xlist:** and **L2** is the **Ylist**. Choose ▫ as the mark. Press [GRAPH] .

Figure 1



[155, 320] scl:15 by [0, 50] scl:5

Figure 2

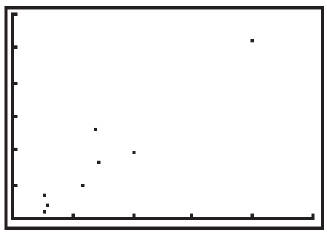


[155, 320] scl:15 by [0, 50] scl:5

Example 5 Scatter Plot, Connected Line Scatter Plot, and Regression Line

a. Use these data to draw a scatter plot: (20.0, 5.2), (10.2, 1.9), (7.3, 1.6), (6.8, 2.6), (5.9, 1.0), (2.6, 0.7), (2.8, 0.35), (2.7, 0.15).

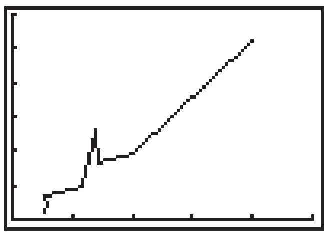
Clear previous data and graphs and set the viewing window. Enter the x-values into **L1** and the y-values into **L2**. Then draw the scatter plot by pressing [2nd] **[STAT PLOT]** 1 [ENTER] [▼] to highlight the scatter plot icon and press [ENTER] . Make sure **L1** is the **Xlist:** and **L2** is the **Ylist:**. Then press [GRAPH] .



[0, 25] scl:5 by [0, 6] scl:1

b. Use the data to draw a line graph.

Press [2nd] **[STAT PLOT]** 1 [▼] [▶] to highlight the line plot icon and press [ENTER] [GRAPH] .

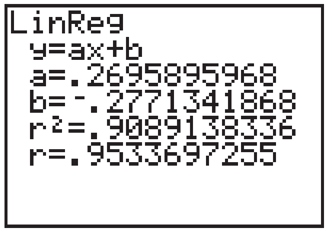


[0, 25] scl:5 by [0, 6] scl:1

c. Draw a regression line for the data in the table.

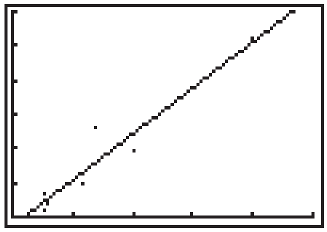
Set the plot to display a scatter plot by pressing [2nd] **[STAT PLOT]** 1 [▼] [ENTER] [2nd] **[QUIT]**.

To calculate the coefficients of regression press [STAT] [▶] 4 [ENTER] .



Then, write the equation of the regression line. You can automatically enter the regression equation in the **Y=** list.

Press [Y=] [VARS] 5 [▶] [▶] 1. Finally, graph the regression line by pressing [GRAPH] .



[0, 25] scl:5 by [0, 6] scl:1

There are also regression models for analyzing data that are not linear built into the calculator.

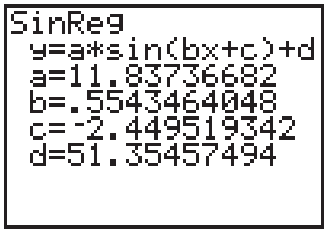
Example 6 **Nonlinear Regression** **Find a sine regression equation to model the data in the table. Graph the data and the regression equation.**

x	1	2	3	4	5	6	7	8	9	10	11	12
y	39	42	45	48	54	59	63	64	59	52	44	40

Enter the data into lists **L1** and **L2**. Press [STAT] 1 [ENTER] 2 [ENTER] … 12 [ENTER] [▶] 39 [ENTER] 42 [ENTER] … 40 [ENTER] .

Find the regression statistics.

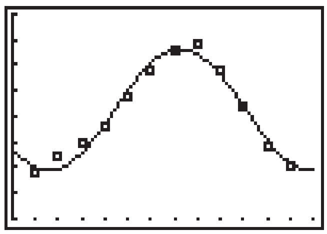
Press: [STAT] [▶] [ALPHA] **[C]** [ENTER]



Enter the regression equation into the **Y=** list.

Press: [Y=] [VARS] 5 [▶] [▶] 1

Then format the scatter plot to graph the data by pressing [2nd] **[STAT PLOT]** 1 [▼] [ENTER] . Make sure that **L1** is chosen as the **Xlist** and **L2** is chosen as the **Ylist**. Set the viewing window. Press [GRAPH] to see the scatter plot and the graph of the regression equation.



[0, 13] scl:1 by [30, 70] scl:5

Also see pages 389, 739, 741–744 for other examples of nonlinear regression.