

# Answers (Lesson 10-2)

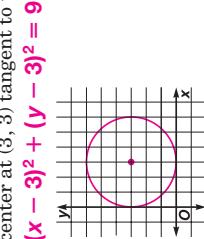
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## Practice

### Circles

**Write the standard form of the equation of each circle described.**  
Then graph the equation.

1. center at  $(3, 3)$  tangent to the  $x$ -axis

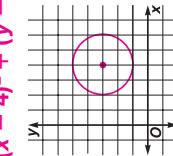


$$(x - 3)^2 + (y - 3)^2 = 9$$

**Write the standard form of each equation. Then graph the equation.**

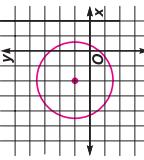
$$3. x^2 + y^2 - 8x - 6y + 21 = 0$$

$$(x - 4)^2 + (y - 3)^2 = 4$$



$$4. 4x^2 + 4y^2 + 16x - 8y - 5 = 0$$

$$(x + 2)^2 + (y - 1)^2 = \frac{25}{4}$$



5.  $(-3, -2), (-2, -3), (-4, -3)$   
 $(x + 3)^2 + (y + 3)^2 = 1;$   
 $(-3, -3); 1$

7. **Geometry** A square inscribed in a circle and centered at the origin has points at  $(2, 2), (-2, 2), (2, -2)$  and  $(-2, -2)$ . What is the equation of the circle that circumscribes the square?

$$x^2 + y^2 = 8$$

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## 10-2

### Spheres

The set of all points in three-dimensional space that are a fixed distance  $r$  (the **radius**), from a fixed point  $C$  (the **center**), is called a **sphere**. The equation below is an algebraic representation of the sphere shown at the right.

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

A line segment containing the center of a sphere and having its endpoints on the sphere is called a **diameter** of the sphere. The endpoints of a diameter are called **poles** of the sphere. A **great circle** of a sphere is the intersection of the sphere and a plane containing the center of the sphere.

1. If  $x^2 + y^2 - 4y + z^2 + 2z - 4 = 0$  is an equation of a sphere and  $(1, 4, -3)$  is one pole of the sphere, find the coordinates of the opposite pole.

$$(-1, 0, 1)$$

2. a. On the coordinate system at the right, sketch the sphere described by the equation  $x^2 + y^2 + z^2 = 9$ .

- b. Is  $P(2, -2, -2)$  inside, outside, or on the sphere?

**outside**

- c. Describe a way to tell if a point with coordinates  $P(a, b, c)$  is inside, outside, or on the sphere with equation  $x^2 + y^2 + z^2 = r^2$ .

**$a^2 + b^2 + c^2 < r^2$ : inside the sphere**  
 **$a^2 + b^2 + c^2 = r^2$ : on the sphere**  
 **$a^2 + b^2 + c^2 > r^2$ : outside the sphere**

3. If  $x^2 + y^2 + z^2 - 4x - 6y - 2z - 2 = 0$  is an equation of a sphere, find the circumference of a great circle, and the surface area and volume of the sphere.

$$\frac{256\pi}{3} \text{ cubic units}$$

4. The equation  $x^2 + y^2 = 4$  represents a set of points in three-dimensional space. Describe that set of points in your own words. Illustrate with a sketch on the coordinate system at the right.

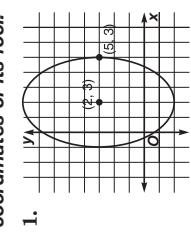
**a cylinder**

# Answers (Lesson 10-3)

## 10-3 Practice

### Ellipses

*Write the equation of each ellipse in standard form. Then find the coordinates of its foci.*

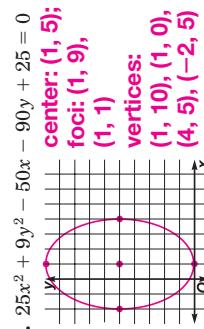


$$\frac{(y-3)^2}{25} + \frac{(x-1)^2}{9} = 1; (1, 7), (1, -1)$$

$$\frac{(x-4)^2}{36} + \frac{(y-2)^2}{16} = 1; (4-2\sqrt{5}, 2), (4+2\sqrt{5}, 2)$$

*For the equation of each ellipse, find the coordinates of the center, foci, and vertices. Then graph the equation.*

$$3. 4x^2 + 9y^2 - 8x - 36y + 4 = 0 \\ \text{center: } (1, 2); \\ \text{foci: } (1 \pm \sqrt{5}, 2) \\ \text{vertices: } (-2, 2), (1, 4), (4, 2), (1, 0)$$



$$4. 25x^2 + 9y^2 - 50x - 90y + 25 = 0 \\ \text{center: } (1, 5); \\ \text{foci: } (1, 9), (1, 1) \\ \text{vertices: } (1, 10), (1, 0), (4, 5), (-2, 5)$$

$$\mathbf{b. } \frac{|x|^2}{3} + \frac{|y|^2}{2} = 1 \quad \mathbf{ellipse}$$

2. In each of the following cases you are given values of  $a$ ,  $b$ , and  $n$  to use in the general equation. Write the resulting equation. Then graph. Sketch each graph on the grid at the right.

$$\mathbf{a. } \frac{|x|^2}{2} + \frac{|y|^2}{3} = 1 \\ a = 2, b = 3, n = 4 \quad \frac{|x|^4}{2} + \frac{|y|^4}{3} = 1$$

$$\mathbf{b. } a = 2, b = 3, n = 6 \quad \frac{|x|^6}{2} + \frac{|y|^6}{3} = 1$$

*Write the equation of the ellipse that meets each set of conditions.*

5. The center is at  $(1, 3)$ , the major axis is parallel to the  $y$ -axis, and one vertex is at  $(1, 8)$ , and  $b = 3$ .  

$$\frac{(y-3)^2}{25} + \frac{(x-1)^2}{9} = 1$$
6. The foci are at  $(-2, 1)$  and  $(-2, -7)$ , and  $a = 5$ .  

$$\frac{(y+3)^2}{25} + \frac{(x+2)^2}{9} = 1$$

3. What shape will the graph of  $\frac{|x|^n}{2} + \frac{|y|^n}{3} = 1$  approximate for greater and greater even, whole-number values of  $n$ ?  
**a rectangle that is 6 units long and 4 units wide, centered at the origin**  
**1.5 ft from the center**

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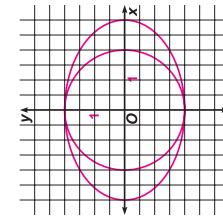
## 10-3 Enrichment

### Superellipses

The circle and the ellipse are members of an interesting family of curves that were first studied by the French physicist and mathematician Gabriel Lamé (1795-1870). The general equation for the family is

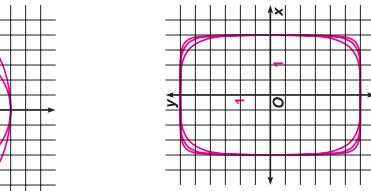
$$\left| \frac{x}{a} \right|^n + \left| \frac{y}{b} \right|^n = 1, \text{ with } a \neq 0, b \neq 0, \text{ and } n > 0.$$

For even values of  $n$  greater than 2, the curves are called superellipses.



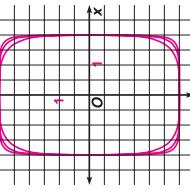
1. Consider two curves that are *not* superellipses. Graph each equation on the grid at the right. State the type of curve produced each time.

$$\mathbf{a. } \frac{|x|^2}{2} + \frac{|y|^2}{2} = 1 \quad \mathbf{circle}$$



2. In each of the following cases you are given values of  $a$ ,  $b$ , and  $n$  to use in the general equation. Write the resulting equation. Then graph. Sketch each graph on the grid at the right.

$$\mathbf{b. } \frac{|x|^2}{3} + \frac{|y|^2}{2} = 1 \quad \mathbf{ellipse}$$



# Answers (Lesson 10-4)

## 10-4

### Practice

#### Hyperbolas

For each equation, find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of its graph. Then graph the equation.

1.  $x^2 - 4y^2 - 4x + 24y - 36 = 0$

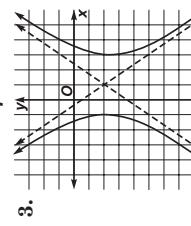


center:  $(0, 0)$ ; foci:  $(\pm 4, 0)$ ;

vertices:  $(\pm 2, 0)$ ;

asymptotes:  $y = \pm \frac{1}{2}(x \mp 4)$

Write the equation of each hyperbola.



$$\frac{(x-0)^2}{9} - \frac{(y+3)^2}{16} = 1$$

5. Write an equation of the hyperbola for which the length of the transverse axis is 8 units, and the foci are at  $(6, 0)$  and  $(-4, 0)$ .

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

6. **Environmental Noise** Two neighbors who live one mile apart hear an explosion while they are talking on the telephone. One neighbor hears the explosion two seconds before the other. If sound travels at 1100 feet per second, determine the equation of the hyperbola on which the explosion was located.

$$\frac{x^2}{1210000} - \frac{y^2}{5759600} = 1$$

## 10-4

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### Enrichment

#### Moving Foci

Recall that the equation of a hyperbola with center at the origin and horizontal transverse axis has the equation  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . The foci are at  $(-c, 0)$  and  $(c, 0)$ , where  $c^2 = a^2 + b^2$ , the vertices are at  $(-a, 0)$  and  $(a, 0)$ , and the asymptotes have equations

$$y = \pm \frac{b}{a}x. \text{ Such a hyperbola is shown at the right.}$$

What happens to the shape of the graph as  $c$  grows very large or very small?

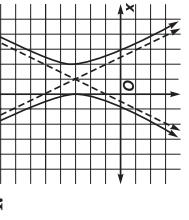
**Refer to the hyperbola described above.**

center:  $(1, 0)$ ; foci:  $(1, \pm 2\sqrt{5})$ ;

vertices:  $(1, \pm 4)$ ;

asymptotes:  $y = \pm 2(x - 1)$

Write the equation of each hyperbola.



$$\frac{(x-1)^2}{1} - \frac{(y-4)^2}{16} = 1$$

5. Write an equation of the hyperbola for which the length of the transverse axis is 8 units, and the foci are at  $(6, 0)$  and  $(-4, 0)$ .

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

6. Use a graphing calculator or computer to graph  $x^2 - y^2 = 1$ ,  $x^2 - y^2 = 0.1$ , and  $x^2 - y^2 = 0.01$ . (Such hyperbolas correspond to smaller and smaller values of  $c$ .) Describe the changes in the graphs. What shape do the graphs approach as  $c$  approaches 0?

**The asymptotes remain the same, but the branches become sharper near the vertices. The graphs approach the lines  $y = x$  and  $y = -x$ .**

7. Suppose  $a$  is held fixed and  $c$  approaches infinity. How does the graph of the hyperbola change?

**The vertices remain at  $(\pm a, 0)$ , but the branches become more vertical. The graphs approach the vertical lines  $x = -a$  and  $x = a$ .**

8. Suppose  $b$  is held fixed and  $c$  approaches infinity. How does the graph of the hyperbola change?

**The vertices recede to infinity and the branches become flatter and farther from the center. As  $c$  approaches infinity, the graphs tend to disappear.**

Recall that the equation of a hyperbola with center at the origin and horizontal transverse axis has the equation  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . The foci are at  $(-c, 0)$  and  $(c, 0)$ , where  $c^2 = a^2 + b^2$ , the vertices are at  $(-a, 0)$  and  $(a, 0)$ , and the asymptotes have equations

$$y = \pm \frac{b}{a}x. \text{ Such a hyperbola is shown at the right.}$$

What happens to the shape of the graph as  $c$  grows very large or very small?

**Refer to the hyperbola described above.**

1. Write a convincing argument to show that as  $c$  approaches 0, the foci, the vertices, and the center of the hyperbola become the same point.
- Since  $0 < a < c$ , as  $c$  approaches 0,  $a$  approaches 0, so the x-coordinates of the foci and vertices approach 0, which is the x-coordinate of the center. Since the y-coordinates are equal, the points become the same.**

2. Use a graphing calculator or computer to graph  $x^2 - y^2 = 1$ ,  $x^2 - y^2 = 0.1$ , and  $x^2 - y^2 = 0.01$ . (Such hyperbolas correspond to smaller and smaller values of  $c$ .) Describe the changes in the graphs. What shape do the graphs approach as  $c$  approaches 0?
- The asymptotes remain the same, but the branches become sharper near the vertices. The graphs approach the lines  $y = x$  and  $y = -x$ .**
3. Suppose  $a$  is held fixed and  $c$  approaches infinity. How does the graph of the hyperbola change?

**The vertices remain at  $(\pm a, 0)$ , but the branches become more vertical. The graphs approach the vertical lines  $x = -a$  and  $x = a$ .**

4. Suppose  $b$  is held fixed and  $c$  approaches infinity. How does the graph of the hyperbola change?

**The vertices recede to infinity and the branches become flatter and farther from the center. As  $c$  approaches infinity, the graphs tend to disappear.**

# Answers (Lesson 10-5)

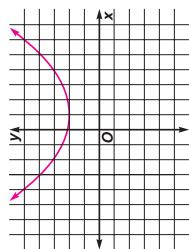
## 10-5 Practice

### Parabolas

For the equation of each parabola, find the coordinates of the vertex and focus, and the equations of the directrix and axis of symmetry. Then graph the equation.

1.  $x^2 - 2x - 8y + 17 = 0$

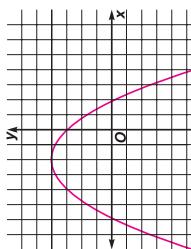
**vertex:**  $(1, 2)$ ; **focus:**  $(1, 4)$ ;  
**directrix:**  $y = 0$ ;  
**axis of symmetry:**  $x = 1$



Write the equation of the parabola that meets each set of conditions. Then graph the equation.

3. The vertex is at  $(-2, 4)$  and the focus is at  $(-2, 3)$ .

$$(x + 2)^2 = -4(y - 4)$$



5. **Satellite Dish** Suppose the receiver in a parabolic dish antenna is 2 feet from the vertex and is located at the focus. Assume that the vertex is at the origin and that the dish is pointed upward. Find an equation that models a cross section of the dish.

$$x^2 = 8y$$

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## 10-5 Enrichment

### Tilted Parabolas

The diagram at the right shows a fixed point  $F(1, 1)$  and a line  $d$  whose equation is  $y = -x - 2$ . If  $P(x, y)$  satisfies the condition that  $PD = PF$ , then  $P$  is on a parabola. Our objective is to find an equation for the tilted parabola, which is the locus of all points that are the same distance from  $(1, 1)$  and the line  $y = -x - 2$ .

To do this, first find an equation for the line  $m$  through  $P(x, y)$  and perpendicular to line  $d$  at  $D(a, b)$ . Using this equation and the equation for line  $d$ , find the coordinates  $(a, b)$  of point  $D$  in terms of  $x$  and  $y$ . Then use  $(PD)^2 = (PF)^2$  to find an equation for the parabola.

Refer to the discussion above.

1. Find an equation for line  $m$ .

$$x - y + (b - a) = 0$$

2. Use the equations for lines  $m$  and  $d$  to show that the coordinates of point  $D$  are  $D(a, b) = D\left(\frac{x-y-2}{2}, \frac{y-x-2}{2}\right)$ .

**From the equation for line  $m$ ,**  

$$-a + b = -x + y$$
. **From the equation for  $d$ ,**  

$$a + b = -2$$
. **Subtract to get  $a = \frac{x-y-2}{2}$ .**  
**Add to get  $b = \frac{y-x-2}{2}$ .**

3. Use the coordinates of  $F$ ,  $P$ , and  $D$ , along with  $(PD)^2 = (PF)^2$  to find an equation of the parabola with focus  $F$  and directrix  $d$ .

$$x^2 - 2xy + y^2 - 8x - 8y = 0$$

4. a. Every parabola has an axis of symmetry. Find an equation for the axis of symmetry of the parabola described above. Justify your answer.

**$y = x$ , since  $y = x$  contains  $F(1, 1)$  and is perpendicular to  $d$ .**

- b. Use your answer from part a to find the coordinates of the vertex of the parabola. Justify your answer.

**$(0, 0)$ , since  $(0, 0)$  is midway between point  $F$  and line  $d$ .**

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<b>10-5 Enrichment</b> <b>Tilted Parabolas</b> <p>The diagram at the right shows a fixed point <math>F(1, 1)</math> and a line <math>d</math> whose equation is <math>y = -x - 2</math>. If <math>P(x, y)</math> satisfies the condition that <math>PD = PF</math>, then <math>P</math> is on a parabola. Our objective is to find an equation for the tilted parabola, which is the locus of all points that are the same distance from <math>(1, 1)</math> and the line <math>y = -x - 2</math>.</p> <p>To do this, first find an equation for the line <math>m</math> through <math>P(x, y)</math> and perpendicular to line <math>d</math> at <math>D(a, b)</math>. Using this equation and the equation for line <math>d</math>, find the coordinates <math>(a, b)</math> of point <math>D</math> in terms of <math>x</math> and <math>y</math>. Then use <math>(PD)^2 = (PF)^2</math> to find an equation for the parabola.</p> <p>Refer to the discussion above.</p> <p>1. Find an equation for line <math>m</math>.</p> $x - y + (b - a) = 0$ <p>2. Use the equations for lines <math>m</math> and <math>d</math> to show that the coordinates of point <math>D</math> are <math>D(a, b) = D\left(\frac{x-y-2}{2}, \frac{y-x-2}{2}\right)</math>.</p> <p><b>From the equation for line <math>m</math>,</b>  <math display="block">-a + b = -x + y</math>. <b>From the equation for <math>d</math>,</b>  <math display="block">a + b = -2</math>. <b>Subtract to get <math>a = \frac{x-y-2}{2}</math>.</b>  <b>Add to get <math>b = \frac{y-x-2}{2}</math>.</b></p> <p>3. Use the coordinates of <math>F</math>, <math>P</math>, and <math>D</math>, along with <math>(PD)^2 = (PF)^2</math> to find an equation of the parabola with focus <math>F</math> and directrix <math>d</math>.</p> $x^2 - 2xy + y^2 - 8x - 8y = 0$ <p>4. a. Every parabola has an axis of symmetry. Find an equation for the axis of symmetry of the parabola described above. Justify your answer.</p> <p><b><math>y = x</math>, since <math>y = x</math> contains <math>F(1, 1)</math> and is perpendicular to <math>d</math>.</b></p> <p>b. Use your answer from part a to find the coordinates of the vertex of the parabola. Justify your answer.</p> <p><b><math>(0, 0)</math>, since <math>(0, 0)</math> is midway between point <math>F</math> and line <math>d</math>.</b></p>				