

4-1 Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial.

1. $6a^4 + a^3 - 2a$ $2; 3a^2 - 7a^5 - 2a^3 + 5$

4; 6

Write a polynomial equation of least degree for each set of roots.

3. 3, -0.5, 1 $x^3 - 7x^2 + 2x + 3 = 0$

4. 3, 3, 1, 1, -2 $x^5 - 6x^4 + 6x^3 + 20x^2 - 39x + 18 = 0$

5. $\pm 2i$, 3, -3 $x^4 - 5x^2 - 36 = 0$

6. $-1, 3 \pm i, 2 \pm 3i$ $x^5 - 9x^4 + 37x^3 - 71x^2 + 12x + 130 = 0$

State the number of complex roots of each equation. Then find the roots and graph the related function.

7. $3x - 5 = 0$ $1; \frac{5}{3}$

8. $x^2 + 4 = 0$ $2; \pm 2i$

9. $c^2 + 2c + 1 = 0$ $2; -1, -1$

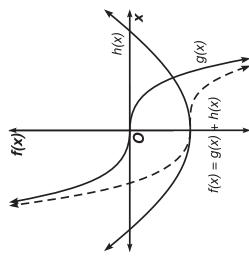
10. $x^3 + 2x^2 - 15x = 0$ $3; -5, 0, 3$

A3

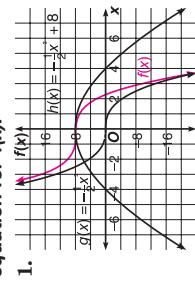
4-1 Enrichment

Graphic Addition

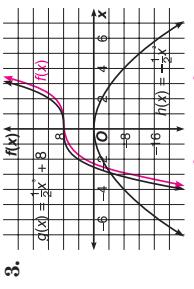
One way to sketch the graphs of some polynomial functions is to use *addition of ordinates*. This method is useful when a polynomial function $f(x)$ can be written as the sum of two other functions, $g(x)$ and $h(x)$, that are easier to graph. Then, each $f(x)$ can be found by mentally adding the corresponding $g(x)$ and $h(x)$. The graph at the right shows how to construct the graph of $f(x) = -\frac{1}{2}x^3 + \frac{1}{2}x^2 - 8$ from the graphs of $g(x) = -\frac{1}{2}x^3$ and $h(x) = \frac{1}{2}x^2 - 8$.



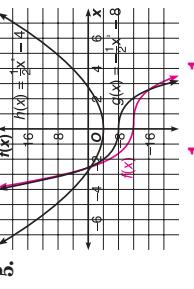
In each problem, the graphs of $g(x)$ and $h(x)$ are shown. Use addition of ordinates to graph a new polynomial function $f(x)$, such that $f(x) = g(x) + h(x)$. Then write the equation for $f(x)$.



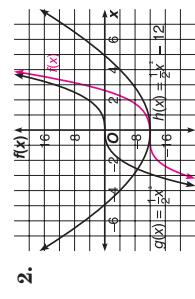
$$f(x) = -\frac{1}{2}x^3 - \frac{1}{2}x^2 + 8$$



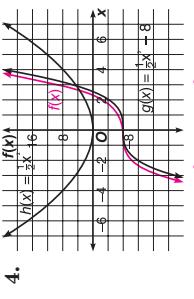
$$f(x) = \frac{1}{2}x^3 - \frac{1}{2}x^2 + 8$$



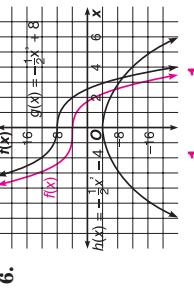
$$f(x) = -\frac{1}{2}x^3 + \frac{1}{2}x^2 - 4$$



$$f(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 - 12$$



$$f(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 + 8$$



$$f(x) = -\frac{1}{2}x^3 + \frac{1}{2}x^2 + 8$$

11. **Real Estate** A developer wants to build homes on a rectangular plot of land 4 kilometers long and 3 kilometers wide. In this part of the city, regulations require a greenbelt of uniform width along two adjacent sides. The greenbelt must be 10 times the area of the development. Find the width of the greenbelt.

8 km

4-3 Practice

The Remainder and Factor Theorems

Divide using synthetic division.

$$1. (3x^2 + 4x - 12) \div (x - 3)$$

3x - 11, R43

$$2. (x^2 - 5x - 12) \div (x - 3)$$

x - 2, R-18

$$3. (x^4 - 3x^2 + 12) \div (x + 1)$$

x³ - x² - 2x + 2, R10

$$4. (2x^3 + 3x^2 - 8x + 3) \div (x + 3)$$

2x² - 3x + 1

Use the Remainder Theorem to find the remainder for each division.
State whether the binomial is a factor of the polynomial.

$$5. (2x^4 + 4x^3 - x^2 + 9) \div (x + 1)$$

0; yes

$$6. (2x^3 - 3x^2 - 10x + 3) \div (x - 3)$$

0; yes

$$7. (3t^3 - 10t^2 + t - 5) \div (t - 4)$$

31; no

$$8. (10x^3 - 11x^2 - 47x + 30) \div (x + 2)$$

0; yes

$$9. (x^4 + 5x^3 - 14x^2) \div (x - 2)$$

0; yes

$$10. (2x^4 + 14x^3 - 2x^2 - 14x) \div (x + 7)$$

0; yes

$$11. (y^3 + y^2 - 10) \div (y + 3)$$

-28; no

$$12. (n^4 - n^3 - 10n^2 + 4n + 24) \div (n + 2)$$

0; yes

$$13. \text{Use synthetic division to find all the factors of } x^3 + 6x^2 - 9x - 54$$

if one of the factors is $x - 3$.

(x - 3)(x + 3)(x + 6)

$$14. \text{Manufacturing} \quad \text{A cylindrical chemical storage tank must have}$$

a height 4 meters greater than the radius of the top of the tank.

Determine the radius of the top and the height of the tank if the tank must have a volume of 15.71 cubic meters.

r ≈ 1 m, h ≈ 5 m

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4-3

Enrichment

The Secret Cubic Equation

You might have supposed that there existed simple formulas for solving higher-degree equations. After all, there is a simple formula for solving quadratic equations. Might there not be formulas for cubics, quartics, and so forth?

There are formulas for some higher-degree equations, but they are certainly not “simple” formulas!

Here is a method for solving a reduced cubic of the form $x^3 + ax + b = 0$ published by Jerome Cardan in 1545. Cardan was given the formula by another mathematician, Tartaglia. Tartaglia made Cardan promise to keep the formula secret, but Cardan published it anyway. He did, however, give Tartaglia the credit for inventing the formula!

Let $R = \left(\frac{1}{2}b\right)^2 + \frac{a^3}{27}$

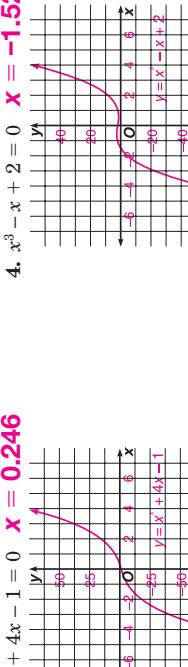
Then, $x = \left[-\frac{1}{2}b + \sqrt{R}\right]^{\frac{1}{3}} + \left[-\frac{1}{2}b - \sqrt{R}\right]^{\frac{1}{3}}$

Use Cardan’s method to find the real root of each cubic equation. Round answers to three decimal places. Then sketch a graph of the corresponding function on the grid provided.

$$1. x^3 + 8x + 3 = 0 \quad x = \textcolor{red}{-0.369}$$



$$2. x^3 - 2x - 5 = 0 \quad x = \textcolor{red}{2.095}$$



$$3. x^3 + 4x - 1 = 0 \quad x = \textcolor{red}{0.246}$$



$$4. x^3 - x + 2 = 0 \quad x = \textcolor{red}{-1.521}$$

